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LIST OF SYMBOLS

A_n, B_n, C_n, ...	Fourier integral coefficients, defined by Equations (36) and (37).
A, B, C, D	Characteristic points, defined in Figure (2).
a	Speed of sound; m/sec.
b	Stream sheet thickness, m.
C_v	Specific heat at constant volume; N•m/kg•K.
H_m, J_m, K_m	Duct response functions.
H	Absolute total enthalpy per unit mass; N•m/kg.
i	$\sqrt{-1}$
i, j	Unit vectors in axial and circumferential directions, respectively.
M	Mach number.
m	Meridional distance; m.
N	Number of grid points along interface of numerical near-field and acoustic far-field solutions.
N_i	Number of blades in the i^{th} row.
Q, R	Boundary conditions at interface of numerical near-field and acoustic far-field solutions.
p	Static pressure, or pressure perturbation; N/m^2.
r	Radial distance from axis of rotation; m.
S	Entropy per unit mass; N•m/kg•K.
S_n	Function defined by Equation (49).
t	Time; sec.
T	Temperature, K.
T_n	Function defined by Equation (61).
U_o	Reference velocity; m/sec.
$\vec{V}$	Absolute velocity or velocity perturbation vector; m/sec.
V_m	Meridional (axial) component of velocity or velocity perturbation; m/sec.
V_θ	Absolute circumferential component of velocity or velocity perturbation; m/sec.
x	Axial distance; m.
y	Circumferential distance; m.
Y	Fundamental wavelength of stage in circumferential direction; m.
α_n, β_n	Acoustic propagation coefficients, defined by Equations (47) and (54); m^{-1}.
$\delta(\tau)$	Dirac delta function (0 for $\tau \neq 0$; 1 for $\tau = 0$).
Δ	Difference operator.
$\Delta(\tau)$	Heaviside step function (0 for $\tau < 0$; 1 for $\tau \geq 0$).

γ	Ratio of specific heats.
Ω	Non-dimensional frequency, defined by Equation (57).
ω	Frequency; sec^{-1}.
ρ	Static density or density perturbation, kg/m^3.
θ	Circumferential angle; radians.
ζ	Radial component of vorticity; sec^{-1}.
∇	Vector operator.

Subscripts:

a, b, c, o	Evaluated at points A, B, C, O.
o	Reference state.
∞	Infinity condition.
i	Inlet station.
d	Discharge station.
m, n	Indices, defined where used.
1	First blade row.
1	Irrotational component of velocity vector.
2	Second blade row.
2	Solenoidal component of velocity vector.

Superscripts:

$\rightarrow$	Vector quantity.
$-$	Time to frequency transform.
o	Spatial location to spatial harmonic transform.
$'$	Perturbation variable.

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III - Acoustic Far-Field Analysis

Simon Slutsky, Dietrich Fischer,
and John I. Erdos

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INTRODUCTION

Under the conditions typically prevalent in highly loaded transonic fan compressor stages, the linearized, small-perturbation approximations to the equations of motion cannot be expected to be descriptive of the flow in the vicinity of the blades. Thus, recourse is made to the numerical solution of the complete nonlinear system of equations, as discussed in Reference (1) in connection with the blade-to-blade program. However, sufficiently far from the blade rows, the amplitude of the flow disturbances will decay to acoustic levels and the linearized, small-perturbation approximations will be descriptive of the far-field. Therefore, an intermediate region in which both analyses are valid should exist at some distance from the blades. The inlet and discharge stations of the blade-to-blade computational domain can serve as the interfaces between the near-field (numerical) and far-field (acoustic) analyses. The present far-field analysis is formulated with respect to an infinite duct model, namely, all outgoing waves should propagate without reflection. It differs, however, from conventional inlet duct analyses in that the signal may begin with an arbitrary transient, associated with the deviation of the assumed initial data in the near-field from the periodic solution which is sought as the asymptotic limit in time. Therefore, the acoustic analysis must recognize that a transient signal will occur during startup and that a simple harmonic time dependence, which is the usual basis of inlet duct acoustics, cannot be assumed. The analysis should allow the transient to radiate outward without reflection, and should be capable of identifying the attainment of a periodic solution by the growth of discrete harmonic components in the solution.

INTERFACE WITH NEAR-FIELD SOLUTION

The procedures for defining inlet and discharge stations in terms of axial boundaries of the computational domains shown in Figure (1) have been described in References (1) and (2). The present analysis pertains to the flow upstream of the inlet station and downstream of the discharge station, where the passage is assumed to be an annular segment of an infinite cylindrical duct. With respect to the analysis of Reference (1), the streamsheet is assumed to have constant radius, r, and thickness, b. However, the circumferential boundaries of the passage are not necessarily restricted to a single blade-to-blade passage

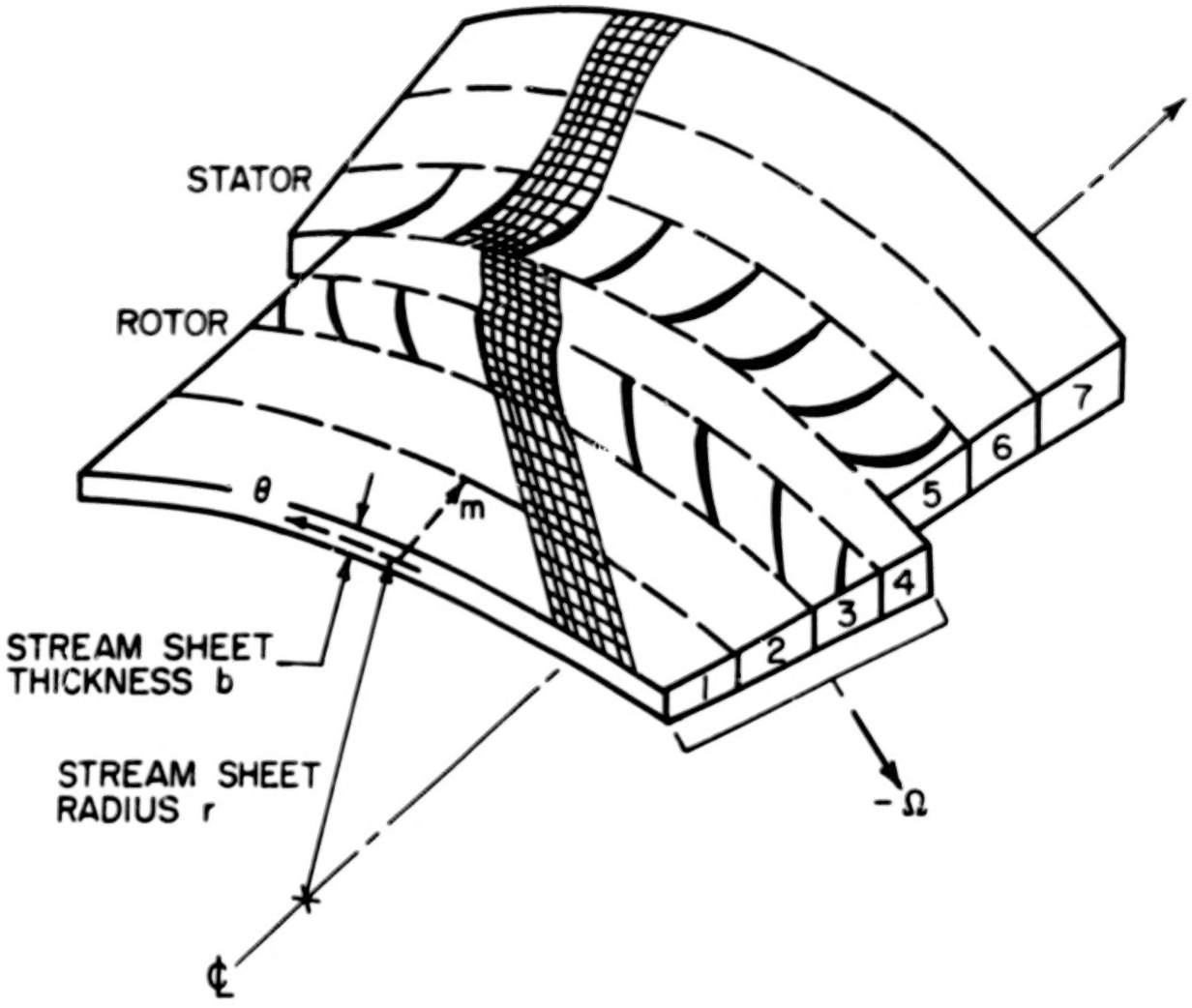


FIGURE 1. BLADE-TO-BLADE COORDINATE SYSTEM AND GRID NETWORK

(as in Reference 1), but should include the angular interval $2\pi/\Delta N$, where $\Delta N = |N_2 - N_1|$. (In the special case $\Delta N = 0$, which is usually avoided in practice, use of a single blade-to-blade passage is again permissible.) Use of the present analysis with the computer code described in Reference (2) will, therefore, require additional storage and retrieval of data along the interfaces between the numerical near-field solution and the present acoustic far-field solution, in the same manner as is performed along the interface between domains 4 and 5 of the near-field solution (see Reference 2).

Derivation of the boundary conditions on the near-field (finite difference) solution, at arbitrary stations referred to as the inlet and discharge boundaries of the near field, was presented in Volume I (Reference 1). The acoustic analysis is formulated in the absolute frame of reference, and accordingly the boundary point analysis from Reference (1) will be restated here in absolute coordinates.

As discussed in Reference (1), the boundary point solution is obtained from a reference plane method-of-characteristics procedure.* A pair of (approximately) two-dimensional wave motion characteristics and a stream surface characteristic are identified at each grid point on the inlet and discharge boundaries, as shown in Figure (2). The compatibility relations,

$$\frac{dp}{dt} \pm \rho a \frac{dV_m}{dt} = - \gamma p \left(\frac{\partial V_\theta}{r\partial\theta} + \frac{V_m}{rb}\frac{drb}{dm}\right) \pm \rho a V_\theta^2 \left(\frac{1}{r}\frac{dr}{dm}\right) \tag{1}$$

apply on the wave motion characteristics,

$$\frac{dm}{V_m \pm a} = \frac{rd\theta}{V_\theta} = dt \tag{2}$$

which are depicted by the lines AO and CO in Figure (2). The equation,

$$S = \text{constant} \tag{3}$$

applies on the stream path characteristic,

$$\frac{dm}{V_m} = \frac{rd\theta}{V_\theta} = dt \tag{4}$$

---

*As pointed out in Reference (1), translation of the reference plane effectively transforms a rotating frame of reference back to an absolute frame. Thus the numerical results are independent of the coordinate system in which the equations are stated.

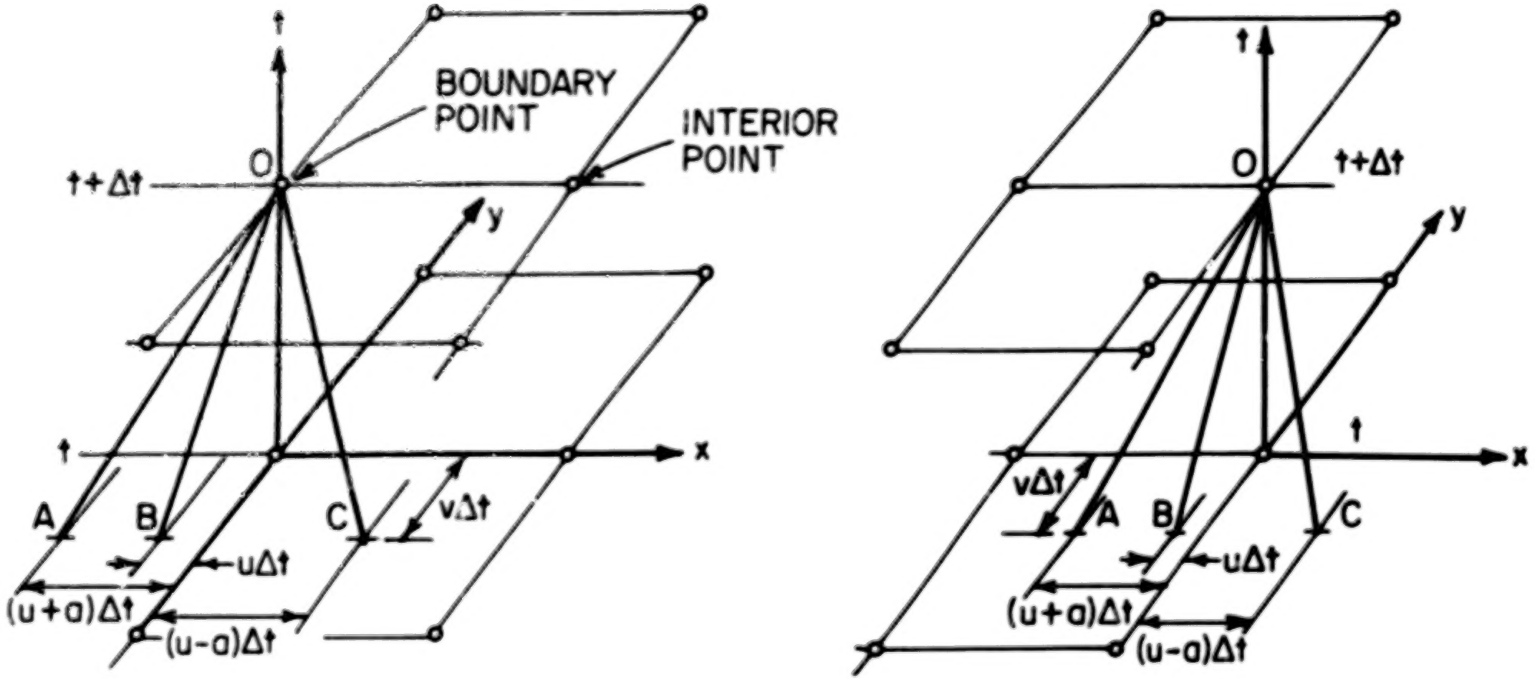


FIGURE 2. CHARACTERISTIC LINES AND GRID POINTS AT INLET AND DISCHARGE STATIONS

which is depicted by the line BO in Figure (2). In addition, if the flow is isentropic

$$\zeta/\rho = \text{constant}$$

also applies on the stream path characteristic, BO. Finally, the circumferential momentum equation is written as:

$$\frac{\partial V_\theta}{\partial t} = - \frac{\partial H}{r\partial\theta} + T\frac{\partial S}{r\partial\theta} - V_m \zeta \tag{5}$$

The inlet flow is assumed to be isentropic and irrotational; thus:

$$S_i = 0, \quad \text{or} \quad p_i/\rho_i^\gamma = p_{-\infty}/\rho_{-\infty}^\gamma \tag{6}$$

and

$$\zeta_i = 0 \tag{7}$$

giving:

$$\left(\frac{\partial V_\theta}{\partial t}\right)_i = - \left(\frac{\partial H}{r\partial\theta}\right)_i \tag{8}$$

Introducing the small disturbance approximations,

$$p' = p - p_{-\infty} << p_{-\infty} \tag{9}$$

$$V_m' = V_m - V_{m_{-\infty}} << V_{m_{-\infty}} \tag{10}$$

$$V_\theta' = V_\theta << V_{m_{-\infty}} \tag{11}$$

etc.

the compatibility equation on the outgoing wave, CO, at the inlet becomes:

$$\begin{aligned} p_i' - \rho_{-\infty} a_{-\infty} V_{m_i}' &= p_c' - \rho_{-\infty} a_{-\infty} V_{m_c}' \\ &- \frac{1}{2}\left(\gamma p_{-\infty}\left(\frac{\partial V_\theta}{r\partial\theta}\right)_i + \gamma p_c \left(\frac{\partial V_\theta}{r\partial\theta} + \frac{V_m}{rb}\frac{drb}{dm}\right)_c\right)\Delta t \\ &- \left(\rho_{-\infty} a_{-\infty} V_{\theta_c}^2 \left(\frac{1}{r}\frac{dr}{dm}\right)_c\right)\Delta t \end{aligned} \tag{12}$$

where the subscript i denotes the inlet value at a new time, t+Δt, i.e., at point 0 in Figure (2), and subscript c denotes the value at point C. Recall that r and b are assumed constant for $m \leq m_i$. Since V_θ can be evaluated at all points 0 along the inlet, from integration of Equation (8), and all variables at point C can be obtained by interpolation of known (current) data in the near-field, the right-hand side can be regarded as a known function of (m_i, θ, t), i.e.:

$$p_i' - \rho_{-\infty} a_{-\infty} V_{m_i}' = \rho_{-\infty} a_{-\infty}^2 Q_i (m_i, \theta, t) \tag{13}$$

$Q_i (m_i, \theta, t)$ is taken as the boundary condition to be placed on the upstream far-field solution. As will be shown in the following section, this condition is sufficient to determine p_i', and thus p_i. V_{m_i}' can then be obtained from Equation (13), and ρ_i from Equation (6). As indicated above, V_{θ_i} is obtained from Equation (8), completing the inlet solution at the new time t+Δt.

Similar procedures at the discharge boundary lead to the relations:

$$p_d/\rho_d^\gamma = p_b/\rho_b^\gamma \tag{14}$$

$$\left(\frac{\partial V_\theta}{\partial t}\right)_d = -\left(\frac{\partial H}{r\partial\theta} - T\frac{\partial S}{r\partial\theta} + V_m \zeta\right)_d \tag{15}$$

$$p_d' + \rho_\infty a_\infty V_{m_d}' = \rho_\infty a_\infty^2 Q_d (m_d, \theta, t) \tag{16}$$

Equation (16) represents one boundary condition on the downstream far-field. Integration of Equation (15) provides a second boundary condition on the downstream far-field, which is required since it cannot be assumed to be irrotational.

$$V_{\theta_d} = a_\infty R_d (m_d, \theta, t) \tag{17}$$

Due to the finite work input by the rotor, the downstream reference conditions (ρ_∞, a_∞) are also not necessarily the same as the upstream reference conditions $(\rho_{-\infty}, a_{-\infty})$. In this case, the far-field analysis provides the value of p_d' and Equations (14), (16) and (17) complete the discharge station solution at the new time t+Δt.

With these preliminaries in hand, attention is now focussed on the acoustic far-field analysis. In the following discussion the primes on the perturbation variables will be dropped as the acoustic solution is understood to pertain only to perturbation quantities. Furthermore, the subscript $(\ )_o$ will be used to denote the reference conditions for either boundary, that is, $x \to \pm \infty$, and all variables without subscript will refer to perturbations with respect to the reference state, for example, $p = p - p_o$ and $\vec{V} = \vec{V} - \vec{V}_o$.

FAR-FIELD ANALYSIS

The problem of interest is the acoustic radiation from the considered blade rows under steady-state operating conditions. However, development of the numerical solution for the near-field includes generation of a transient flow field associated with the deviation of the assumed initial conditions from a periodic solution, which is sought as an asymptotic limit in time. Therefore, analysis of the far-field must necessarily recognize that a transient signal will occur during startup and that a simple harmonic time dependence, which is the usual basis of engine acoustic analyses, cannot be assumed to be descriptive until the asymptotic limit is approached. The analysis should allow the transient to radiate outward, without reflection, and should be capable of identifying the attainment of a periodic solution by the growth of discrete harmonic components.

Use will be made of a harmonic type analysis to develop an acoustic formulation of the pressure field in which frequency effects are superimposed and the time history obtained by integration. This procedure leads to a convolution integral (i.e., a type of Duhamel integral) in the time domain, which is more directly compatible with the numerical data in the near-field than a result in the frequency domain. (The near-field data consists of values of the pressure, flow velocity, etc., at a discrete series of grid points and at fixed time intervals.)

Since the present effort is addressed toward a cascade formulation, the governing equations are written in a two-dimensional, Cartesian coordinate system. It is noted that the two-dimensional problem could be described by a solution of the wave equation alone, were it not for the fact that the time dependent force distribution on the blades is capable of producing a convected

vorticity field (as well as a corresponding entropy field, which, however, is not relevant to the present problem). It will be seen that the convected vorticity field does not contribute to the acoustic pressure field, per se, as distinguished from the irrotational component of the velocity field which is directly coupled with the acoustic pressure. Therefore, the velocity perturbation field downstream of the discharge boundary and upstream of the inlet boundary (see Figure 1) is characterized by the sum of an irrotational ($\nabla x=0$) velocity vector $\vec{V}_1$ and a solenoidal ($\nabla\cdot=0$) velocity vector $\vec{V}_2$, which satisfy the linearized conservation equations. Note that the inlet flow has been assumed to be irrotational, therefore, $\vec{V}_2=0$ in the upstream far-field; however, the analysis will be developed with respect to the more general case pertaining to the downstream far-field.

The linearized conservation equations for the perturbation variables can be stated as*

$$\frac{dp}{dt} + \rho_o a_o^2 \nabla\cdot\vec{V} = 0 \tag{18}$$

$$\rho_o \frac{d\vec{V}}{dt} + \nabla p = 0 \tag{19}$$

$$\frac{dp}{dt} - a_o^2 \frac{d\rho}{dt} = \frac{P_o}{C_v}\frac{dS}{dt} \tag{20}$$

where:

$$\vec{V} = \vec{V}_1 + \vec{V}_2 \tag{21}$$

$$\vec{V}_1 = \vec{i}\, V_{m_1} + \vec{j}\, V_{\theta_1} \tag{22}$$

$$\vec{V}_2 = \vec{i}\, V_{m_2} + \vec{j}\, V_{\theta_2} \tag{23}$$

$$\nabla \times \vec{V}_1 = 0 \tag{24}$$

$$\nabla \times \vec{V}_2 = \zeta \neq 0 \tag{25}$$

---

*These represent linearized versions of Equations (13), (14), (15) and (17) of Reference (1), with r = constant and b = constant.

$$\nabla \cdot \vec{V}_2 = 0 \tag{26}$$

$$U_o = M_o a_o \tag{27}$$

and

$$\frac{d}{dt} = \frac{\partial}{\partial t} + U_o \frac{\partial}{\partial m} \tag{28}$$

Since the streamsheet has been assumed to be cylindrical in the far-field, with $b << r$, the conventional Cartesian coordinates (x,y) will be used in place of the meridional coordinates (m,θ):

$$x = m \tag{29a}$$

$$y = r\theta \tag{29b}$$

Substituting Equation (26) into (18) gives:

$$\frac{dp}{dt} + \rho_o a_o^2 \nabla \cdot \vec{V}_1 = 0 \tag{30}$$

Substituting Equations (24) and (26) into the curl and divergence of Equation (19), respectively, gives:

$$\frac{d}{dt} (\nabla \cdot \vec{V}_1) + \frac{1}{\rho_o} \nabla^2 p = 0 \tag{31}$$

and:

$$\frac{d}{dt} (\nabla \times \vec{V}_2) = 0 \tag{32}$$

Equations (18) and (19) or (30) and (31) yield the wave equation for the pressure:

$$\frac{1}{a_o^2} \frac{d^2 p}{dt^2} - \nabla^2 p = 0 \tag{33}$$

Therefore, solutions for p are purely radiative. Since Equations (30) and (31) can be also combined to yield the wave equation, solutions for $\vec{V}_1$ are also purely radiative. However, Equation (32) infers that solutions for $\vec{V}_2$ are purely convective. Therefore:

$$\frac{d\vec{V}_2}{dt} = 0 \tag{34}$$

and:

$$\frac{d\vec{V}_1}{dt} + \frac{1}{\rho_o} \nabla p = 0 \tag{35}$$

Solutions to Equations (33), (34) and (35) can be expressed in terms of a Fourier integral representation:

$$\begin{Bmatrix} p(x,y,t) \\ V_{m_1}(x,y,t) \\ V_{\theta_1}(x,y,t) \end{Bmatrix} = \sum_n e^{-i\alpha_n y} \int_{-\infty}^{\infty} \begin{Bmatrix} \rho_o\, a_o\, A_n(\omega) \\ B_n(\omega) \\ C_n(\omega) \end{Bmatrix} a_o e^{i(\omega t - \beta_n x)} \frac{d\omega}{2\pi} \tag{36}$$

$$\begin{Bmatrix} V_{m_2}(x,y,t) \\ V_{\theta_2}(x,y,t) \end{Bmatrix} = \sum_n e^{-i\alpha_n y} \int_{-\infty}^{\infty} \begin{Bmatrix} D_n(\omega) \\ E_n(\omega) \end{Bmatrix} a_o e^{i\omega(t - x/U_o)} \frac{d\omega}{2\pi} \tag{37}$$

Consider first the downstream boundary on which (see Equations 16 and 17):

$$Q_d(y,t) = (p_d + \rho_o\, a_o\, V_{m_d})/(\rho_o\, a_o^2) \tag{38}$$

$$R_d(y,t) = V_{\theta_d}/a_o \tag{39}$$

Thus, the boundary data can be expressed as:

$$\begin{Bmatrix} Q(y,t) \\ R(y,t) \end{Bmatrix} = \sum_n e^{-i\alpha_n y} \int_{-\infty}^{\infty} \begin{Bmatrix} \bar{Q}_n^o(\omega) \\ \bar{R}_n^o(\omega) \end{Bmatrix} e^{i\omega t} \frac{d\omega}{2\pi} \tag{40}$$

where $\bar{(\ )}$ denotes the time to frequency transform, as above, while $(\ )^o$ indicates transformation from spatial location, y, to spatial harmonic, n, which will be discussed later (cf. Equation 55). (The subscript d has been dropped since the ensuing development can apply to either the inlet or discharge boundary.)

Substitution of the above integral forms into Equation (24), (26), (32), (34), (38) and (39) gives:

$$C_n = \frac{\alpha_n}{\beta_n} B_n \tag{41}$$

$$E_n = - \frac{\omega}{U_o \alpha_n} D_n \tag{42}$$

$$B_n = \frac{a_o \beta_n}{\omega - U_o \beta_n} A_n \tag{43}$$

$$A_n + B_n + D_n = \bar{Q}_n^o \tag{44}$$

$$C_n + E_n = \bar{R}_n^o \tag{45}$$

The value of A_n, and thus B_n, C_n, D_n and E_n, can thereby be expressed in terms of the transform of the boundary data, viz.:

$$A_n = \frac{\omega - U_o \beta_n}{\omega^2 + (a_o - U_o)\beta_n \omega + U_o a_o \alpha_o^2} \left[\omega \bar{Q}_n^o + U_o \alpha_n \bar{R}_n^o\right] \tag{46}$$

The propagation coefficient β_n is determined by substituting the integral relation for pressure (Equation 36) into the wave equation (Equation 33):

$$\beta_n = \frac{-\omega M_o \pm \left(\omega^2 - \alpha_n^2 a_o^2 (1-M_o^2)\right)^{\frac{1}{2}}}{a_o (1-M_o^2)} \tag{47}$$

where the + sign refers to downstream propagation (from the discharge boundary) and the - sign to upstream propagation (from the inlet boundary).

Equations (46) can now be substituted into the corresponding integral relation (Equation 36) to obtain, after some manipulation, the pressure perturbation on the discharge boundary in terms of the specified boundary data.

$$p_d(y,t) = \frac{\rho_o a_o}{(1-M_o^2)} \sum_n \frac{e^{-i\alpha_n y}}{\alpha_n a_o} \int_{-\infty}^{\infty} (\omega - S_n) \left(\frac{\omega}{\alpha_n a_o} \bar{Q}_n^o + M_o \bar{R}_n^o\right) e^{i\omega t} \frac{d\omega}{2\pi} \tag{48}$$

where

$$S_n = (\omega^2 - \alpha_n^2 a_o^2 (1-M_o^2))^{\frac{1}{2}} \tag{49}$$

Consider now the inlet boundary where $\vec{V}_2 = 0$. The boundary condition here is stated as (see Equation 13):

$$Q_i(y,t) = (p_i - \rho_o a_o V_{m_i})/(\rho_o a_o^2) \tag{50}$$

Since $\vec{V}_2 = 0$, $D_n = E_n = 0$, and $\bar{R}_n^o$ becomes part of the solution, i.e., it cannot be specified as a boundary condition, consistent with the near field analysis described in the previous section. Thus, A_n is only a function of Q_i, viz:

$$A_n = \frac{\omega - U_o \beta_A}{\omega - (a_o + U_o)\beta_n} \bar{Q}_n^o \tag{51}$$

where

$$Q_i(y,t) = \sum_n e^{-i\alpha_n y} \int_{-\infty}^{\infty} \bar{Q}_n^o(\omega)\, e^{i\omega t}\, \frac{d\omega}{2\pi} \tag{52}$$

Again, with some manipulation, an equation for the pressure perturbation on the inlet boundary is obtained in terms of the specified data:

$$p_i(y,t) = \frac{\rho_o a_o^2}{(1-M_o^2)(1+M_o)} \sum_n \frac{e^{-i\alpha_n y}}{\alpha_n^2 a_o^2} \int_{-\infty}^{\infty} (\omega - S_n)(\omega + M_o S_n) \bar{Q}_n^o\, e^{i\omega t}\, \frac{d\omega}{2\pi} \tag{53}$$

The selected representation of the solutions, Equations (36) and (37), and boundary conditions, Equations (40) and (52), as Fourier series in the y direction is appropriate for enforcement of the periodicity boundary condition per-

taining to spatial variations in this direction. The coefficient α_n is defined accordingly:

$$\alpha_n = \frac{2\pi n}{Y} = \frac{n\Delta N}{r} \tag{54}$$

where Y is the fundamental wave length of the stage (double cascade) configuration (i.e, $Y = 2\pi r/\Delta N$). In addition, the fact that the boundary data is specified at a discrete number of grid points, say N, on the boundaries implies that the Fourier series can only include N terms; n=0, 1, 2...N-1. Since the distance y to each point can be written as mY/N, where m=0, 1, 2...N-1 also, the Fourier series can be expressed in the standard Discrete Fourier Transform (DFT) notation:

$$\begin{Bmatrix} Q(y,t) \\ R(y,t) \end{Bmatrix} = \begin{Bmatrix} Q_m(t) \\ R_m(t) \end{Bmatrix} = \sum_{n=0}^{N-1} \begin{Bmatrix} Q_n^o(t) \\ R_n^o(t) \end{Bmatrix} e^{-\frac{2\pi inm}{N}} \tag{55}$$

The inverse DFT is then:

$$\begin{Bmatrix} Q_n^o(t) \\ R_n^o(t) \end{Bmatrix} = \frac{1}{N} \sum_{m=0}^{N-1} \begin{Bmatrix} Q_m(t) \\ R_m(t) \end{Bmatrix} e^{2\pi inm/N} \tag{56}$$

For computational purposes, the following non-dimensional parameters are evident in Equations (48) and (53):

$$\Omega = \omega/\alpha a_o \tag{57}$$

$$\tau = \alpha a_o t \tag{58}$$

where

$$\alpha = 2\pi/Y = \alpha_n/n \tag{59}$$

Then, on the discharge boundary:

$$p_n^o(\tau) = \frac{\rho_o a_o^2}{(1-M_o^2)n^2} \int_{-\infty}^{\infty} (\Omega - T_n)\ (\Omega\ \bar{Q}_n^o(\Omega) + n\ M_o\ \bar{R}_n^o\ (\Omega) e^{i\Omega\tau})\ \frac{d\Omega}{2\pi} \tag{60}$$

where:

$$T_n = (\Omega^2 - (1-M_o^2)n^2)^{\frac{1}{2}} \tag{61}$$

$$\bar{Q}_n^o(\Omega) = \alpha a_o\ \bar{Q}_n^o(\omega) \tag{62}$$

$$\bar{R}_n^o(\Omega) = \alpha a_o\ \bar{Q}_n^o(\omega) \tag{63}$$

and on the inlet boundary:

$$p_n^o(\tau) = \frac{\rho_o a_o^2}{(1-M_o^2)\ (1+M_o)n^2} \int_{-\infty}^{\infty} (\Omega - T_n)\ (\Omega + M_o T_n)\ \bar{Q}_n^o(\Omega) e^{i\Omega\tau}\ \frac{d\Omega}{2\pi} \tag{64}$$

Then, on either boundary:

$$p_m(\tau) = \sum_{n=0}^{N-1} e^{-2\pi i n m/N}\ p_n^o(\tau) \tag{65}$$

The integrals of Equations (60) and (64) can be transformed back to the time domain by the following convolution. On the discharge boundary:

$$p_n^o(\tau) = \rho_o a_o^2\ (H_n^o(\tau)\ \ast\ Q_n^o(\tau) + J_n^o(\tau)\ \ast\ R_n^o(\tau)) \tag{66}$$

where $\ast$ denotes convolution in time. For $n \geq 1$:

$$H_n^o(\tau) = \frac{1}{n^2(1-M_o^2)} \int_{-\infty}^{\infty} \Omega\ (\Omega - T_n)\ e^{i\Omega\tau}\ \frac{d\Omega}{2\pi} \tag{67}$$

$$J_n^o(\tau) = \frac{1}{n(1-M_o^2)} \int_{-\infty}^{\infty} (\Omega - T_n)\, e^{i\Omega\tau}\, \frac{d\Omega}{2\pi} \tag{68}$$

and, for n=0:

$$H_o^o(\tau) = \frac{1}{2}\, \delta(\tau) \tag{69}$$

$$J_o^o(\tau) = 0 \tag{70}$$

$$p_o^o(\tau) = \frac{1}{2}\, \rho_o a_o^2 Q_o^o(\tau) \tag{71}$$

where δ is the Dirac delta function.

Note that for $\tau \to 0$ with $n \neq 0$:

$$H_n^o(\tau) \to \frac{1}{2}\, \delta(\tau) \tag{72}$$

$$p_n^o(\tau) \to \frac{1}{2}\, \rho_o a_o^2 Q_n^o(\tau) \tag{73}$$

On the inlet boundarcy:

$$p_n^o(\tau) = \rho_o a_o^2 K_n^o(\tau) \equiv p_n^o(\tau) \tag{74}$$

where, for $n \geq 1$

$$K_n^o(\tau) = \frac{1}{n^2(1-M_o^2)\,(1-M_o^2)} \int_{-\infty}^{\infty} (\Omega - T_n)\,(\Omega + M_o T_n)\, e^{i\Omega\tau}\, \frac{d\Omega}{2\pi} \tag{75}$$

and, for n=0:

$$K_o^o(\tau) = \frac{1}{2}\,\delta(\tau) \tag{76}$$

$$p_o^o(\tau) = \frac{1}{2}\,\rho_o a_o^2 p_o^o(\tau) \tag{77}$$

while for $t \to 0$ with $n \neq 0$:

$$k_n^o(\tau) = \frac{1}{2}\,\delta(\tau) \tag{78}$$

$$p_n^o(\tau) = \frac{1}{2}\,\rho_o a_o^2 p_n^o(\tau) \tag{79}$$

The integral in Equation (67) is evaluated by dividing the non-dimensional frequency range into three regions, viz: $-\infty$ to $-\Omega_o$, $-\Omega_o$ to Ω_o, and Ω_o to ∞. In the middle region standard DFT techniques are applied, while in the outer regions, $\Omega \to \pm\infty$, the integral is accurately approximated in terms of an exponential integral of the form

$$\int_{\pm\Omega_o}^{\pm\infty} \frac{e^{i\Omega\tau}}{\Omega^{\alpha}}\, d\Omega \tag{80}$$

for which closed form expressions are available. The integral in Equation (68) can be expressed in terms of a Bessel function:

$$\int_{-\infty}^{\infty} e^{i\Omega\tau}\,\left(\Omega - (\Omega^2 - c^2)^{\frac{1}{2}}\right)\,\frac{d\Omega}{2\pi} = ic^2\,\frac{J_1(c\tau)}{c\tau}\,\Delta(\tau) \tag{81}$$

where c denotes $n(1-M_o^2)^{\frac{1}{2}}$ and $\Delta(\tau)$ is the Heaviside step function. The evaluation of the integral in Equation (75) is carried out in the same fashion as that in Equation (67).

The desired solution for the pressure perturbation on the interface with near-field is finally accomplished by observing that Equation (65) can now be expressed in the form:

$$p_m(t) = \rho_o a_o^2 \sum_{n=0}^{N-1} e^{-2\pi inm/N} \left\{ \begin{array}{l} H_n^o(t) * Q_n^o(t) + J_n^o(t) * R_n^o(t) \\ K_n^o(t) * Q_n^o(t) \end{array} \right\} \quad (82)$$

on the discharge and inlet boundaries respectively. Use of Equation (55) for $P_m(t)$, $Q_m(t)$ and $R_m(t)$ and similar DFT expansions for H_m, J_m and K_m then leads to the convolution:

$$p_m(t) = \frac{\rho_o a_o^2}{N} \sum_{n=0}^{N-1} \left\{ \begin{array}{l} H_n(t) * Q_{m-n}(t) + J_n(t) * R_{m-n}(t) \\ K_n(t) * Q_{m-n}(t) \end{array} \right\} \quad (83)$$

on the discharge and inlet boundaries respectively. Thus, a double convolution over both time and distance (in the y direction) is required. The functions* $Q_m(t)$ and $R_m(t)$, therefore, represent the point sources of time-varying strength which are aligned along the considered boundaries, with spatial resolutions consistent with the number of grid points specified.

The functions $H_m(t)$, $J_m(t)$ and $K_m(t)$ are the duct response functions, defined by:

---

*Bear in mind that $Q_m(t)$ denotes either $Q_i(y_m,t)$ at the inlet station or $Q_d(y_m,t)$ at the discharge station, while $R_m(t)$ denotes $R_d(y_m,t)$.

$$\begin{Bmatrix} H_m(t) \\ J_m(t) \\ K_m(t) \end{Bmatrix} = \sum_{n=0}^{N-1} \begin{Bmatrix} H_n^o(t) \\ J_n^o(t) \\ K_n^o(t) \end{Bmatrix} e^{-2\pi inm/N}$$

$$= \begin{Bmatrix} \frac{1}{2}\delta(t) \\ 0 \\ \frac{1}{2}\delta(t) \end{Bmatrix} + \sum_{n=1}^{N-1} \begin{Bmatrix} H_n^o(t) \\ J_n^o(t) \\ K_n^o(t) \end{Bmatrix} e^{-2\pi inm/N} \qquad (84)$$

Finally, it should be noted that only the real part of $p_m(t)$ is of interest. Equations (67), (68) and (75) infer that $H_n^o(\tau)$ and $K_n^o(\tau)$ are purely real functions and $J_n^o(\tau)$ is a pure imaginary function. The boundary data $Q_m(t)$ and $R_m(t)$ are obviously all real numbers. Therefore, it is concluded that

$$J_m(t) = \sum_n - i\, J_n^o(t) \sin \frac{2\pi nm}{N} = \sum_n L_n^o(t) \sin \frac{2\pi nm}{N} \qquad (85)$$

where

$$L_n^o(\tau) = \frac{1}{\pi(1-M_o^2)} \int_{-\infty}^{\infty} (\Omega - T_n) e^{i\Omega\tau} \frac{d\Omega}{2\pi} \qquad (86)$$

i.e., the solution can be carried out entirely in terms of real functions.

NUMERICAL EXAMPLE

A group of subroutines including an efficient Fast Fourier Transform routine has been developed, for use in conjunction with the blade-to-blade computer program B2DATL, described in Reference (2), to carry out the indicated convolutions required to solve Equation (83) numerically. Results have been thus far limited to test cases with a simple harmonic input signal. For

example, in one of the calculations the discharge station was assumed to be divided into 8 intervals covering a total circumferential distance of 0.1 m. The selected reference (average) Mach number of the discharge flow was 0.8. The flow was assumed to be irrotational so that only one input function Q(y,t) was required, and the second function R(y,t) could be considered as a response function (that is, it was calculated from Q(y,t)). The input function $Q(y,t) = \cos(\Omega\tau - 2\pi n/N)$, with $Q(y,t) = 0$ for $\tau < 0$, was selected for this case, where $\tau = 2\pi N a_o t/Y$, $(n = 0, 1, 2, \ldots, N-1)$ $N = 8$, $Y = 0.1$ m., $a_o = 10^3$ m/sec, and $\Omega = 1$. The input function Q and response function R at $n = 0$ are plotted in Figure (3). It should be noted that the response function is initially out of phase with the input function because of the assumption that $Q = 0$ for $\tau < 0$. However, the effect of the transient at $\tau = 0$ dies quickly, and after about 1/3 millisecond the response function closely approximates the input function and indicates the desired harmonic solution is being approached asymptotically. The non-dimensional perturbation pressure is plotted in Figure (4). The complete history is shown for the point $n = 0$, whereas the history of the points $n = 1$ and 2 is only shown at early times, where a difference in amplitude as well as phase exists. The values of p_1 and p_2 are not plotted at later times (that is, after about 1/3 millisecond) since the pressure solutions at the various grid points only differ noticeably in the phase angle corresponding to the input function and the differences in amplitude asymptotially decay. However, the complete numerical results for $n = 0, 1, 2 \ldots 7$ are included in Appendix B as the first test case.

DESCRIPTION OF COMPUTER CODE

Overview

Program RAFFT (Real Acoustic Far-Field Theory), listed in Appendix A, is designed to compute the real part of the acoustic far-field of a pair of interacting cascades in an infinite duct. This program has not as yet been coupled to program B2DATL which computes the near field of the cascades. It currently consists of two parts. The first part, which is made up of the main program RAFFT, the subroutine GETQR (which calculates the input functions Q and R) and the function routine TRØØT, is at present used solely for the test purposes; it must be replaced by a calling routine in B2DATL which calculates the input

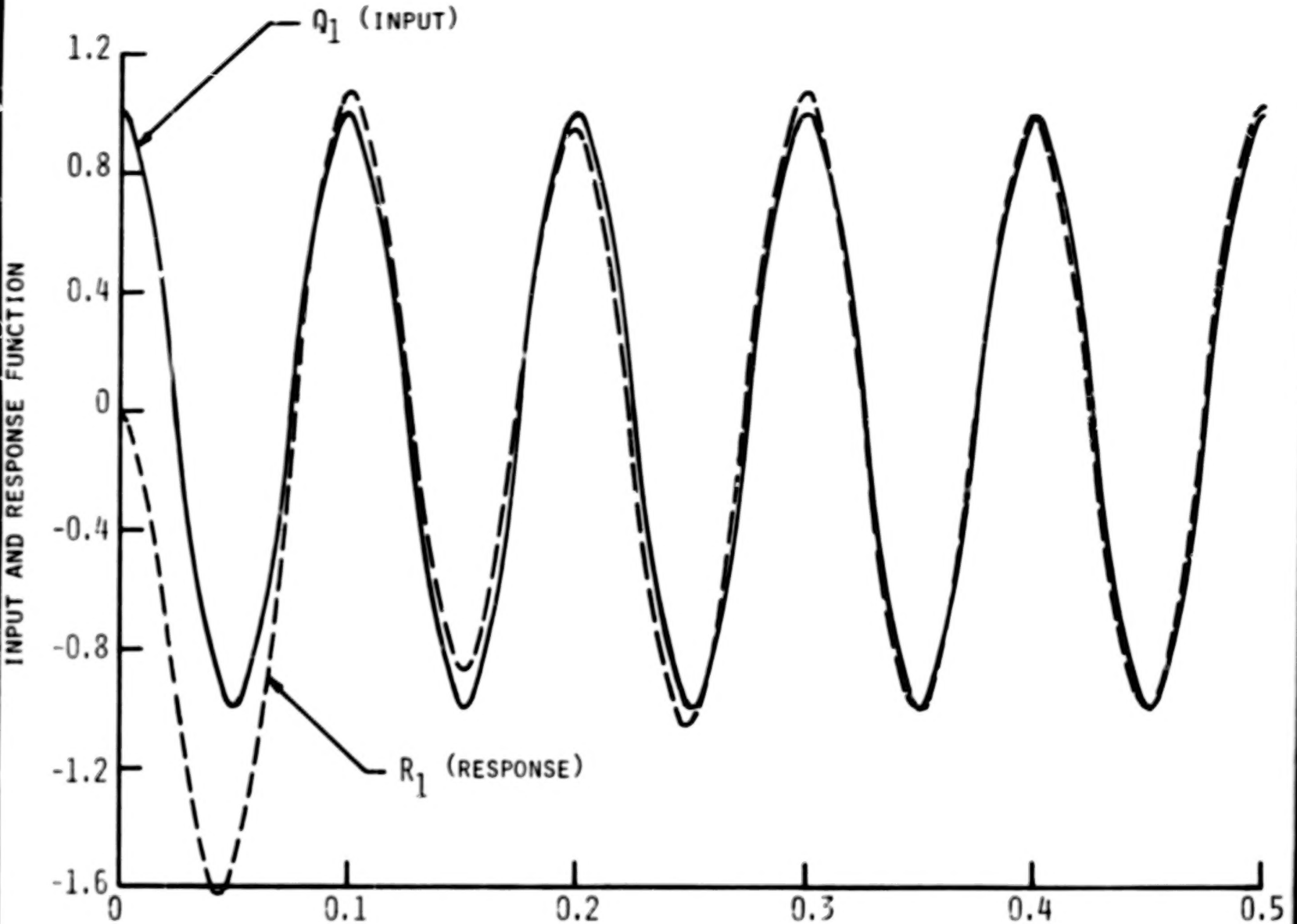


FIGURE 3. INPUT AND RESPONSE FUNCTIONS FOR TEST CASES.

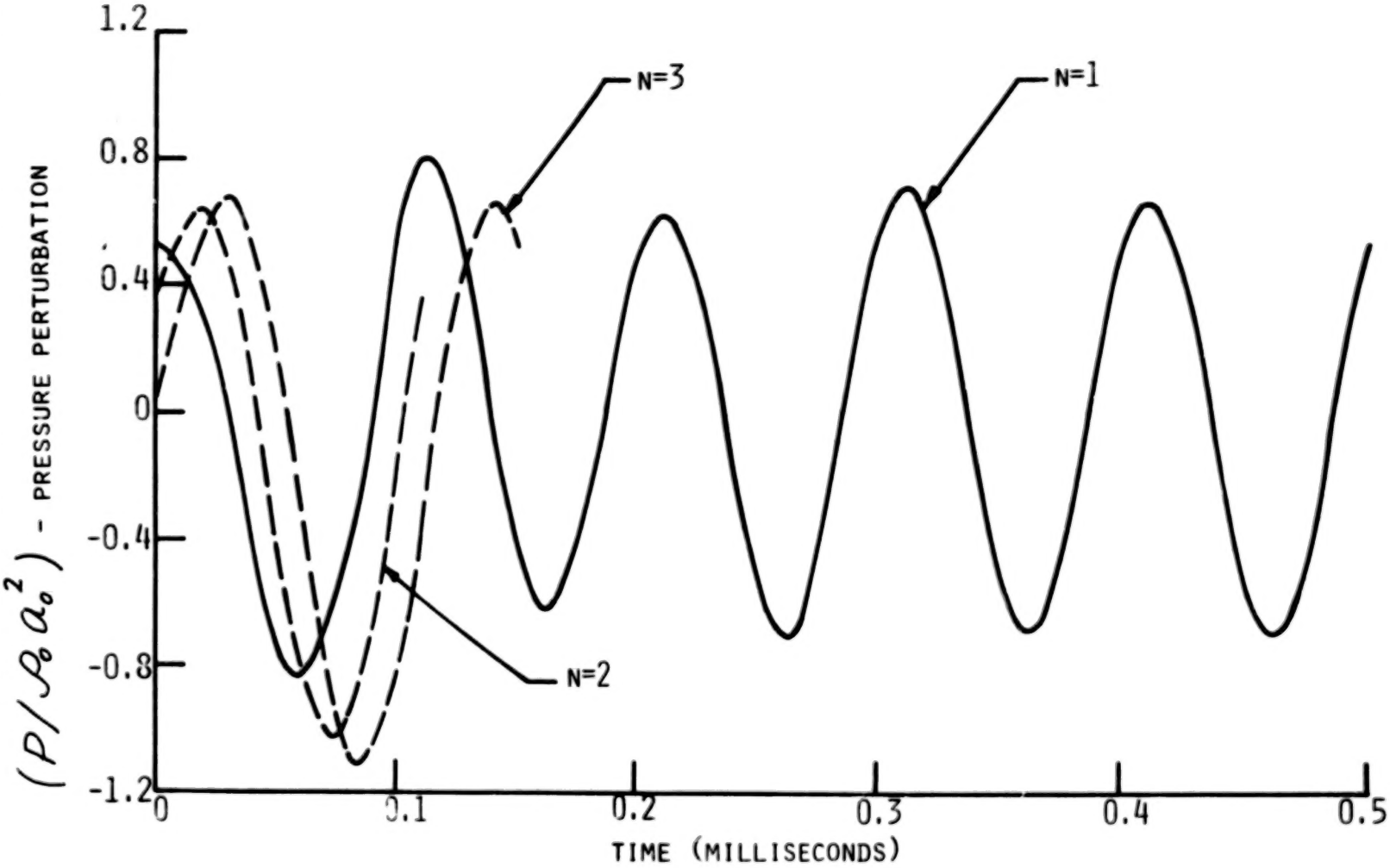


FIGURE 4. PRESSURE PERTURBATIONS AT SEVERAL GRID POINTS

functions Q and R from the boundary data in accord with Equations (38), (39) and (50). The remainder of the program consists of seven subroutines and function routines: AJ1DX, CØNVØL, FFT, FFT2, GREENS, PØLY and TCØNV. Only subroutine CØNVØL is called from the main program; all other subroutines and function routines are called from CØNVØL. Subroutine CØNVØL is called by the statement:

```
CALL CØNVØL (P, Q, R, NY, IT, DTIME, MAXCØN, CMARK, RHØO, C, Y, FIRST,
             NDIMY, NDIMT, PU)
```

which requires all variables as input parameters except P and PU which are the only results. P(IY, IT) is the discharge station perturbation pressure, as a function of discrete spatial location space, IY and time, IT, and PU(IY,IT) is the corresponding inlet pressure.

Input Data

The input parameters Q(IY,IT), R(IY,IT), NY, IT, DTIME, MAXCØN, CMARK, RHØO, CY, Y, FIRST, NDIMY and NDIMT, required for the evaluation of P(IY,IT) through the statement CALL CØNVØL (...), are defined as follows:

NAME	DIMENSION	DEFINITION
Q	(8,1001)	Near-field discharge or inlet boundary condition, $(p' \pm \rho_o a_o V_m')/\rho_o a_o^2$. See Equations (38) and (50).
R	(8,1001)	Circumferential perturbation velocity, V_θ'/a_o. See Equation (39).
NY		Number of grid points in y-direction.
IT		Current index.
DTIME		Time step; seconds.
MAXCØN		Maximum number of points used for convolution in time ($\leq$ 1001, with present dimension of Q and R).
CMARK		Reference Mach number, M_o.
RHØO		Reference density, ρ_o; kg/m^3.
C		Reference speed of sound, a_o; m/sec.

NAME	DIMENSION	DEFINITION
Y		Fundamental wavelength of the cascade ($2\pi r/\Delta N$); m. See Equation (54).
ØMEGAO		Input frequency (non-dimensional), Ω. See Equation (57).
FIRST		A logical parameter which must be set to "FIRST = .TRUE." before subroutine CØNVØL is called for the first time. This avoids repetition of calculations that need to be done only once. Subroutine CØNVØL automatically sets "FIRST=.FALSE." after the initial call.
NDIMY		Dimension of the arrays Q and R in the Y (circumferential) direction, presently 8. (NY $\leq$ NDIMY is necessary.)
NDIMT		Dimension of the arrays Q and R in time, presently 1001.

Subroutine and Function Sub-Programs

The following seven subroutines and function routines encompass the main portion of the program necessary to determine the acoustic far-field solution:

NAME	DESCRIPTION
CØNVØL	When called the first time, CØNVØL calculates a number of parameters used through the rest of the program, calls subroutine GREENS to calculate the Green's functions H_m and J_m. Thereafter, it calls subroutine TCØNV to perform the convolutions, and finally it evaluates Equations (83): $$P(IY,IT)=\frac{RH\emptyset O\cdot C\cdot C}{NY}\left(\sum_{KY=1}^{NY}[Q(IY,IT)*H(KY,IT)+R(IY,IT)*J(KY,IT)]\right)$$ where * denotes convolution in time.
AJ1DX	Evaluates the Bessel function J1(X)/X using a 7th order polynomial approximation.

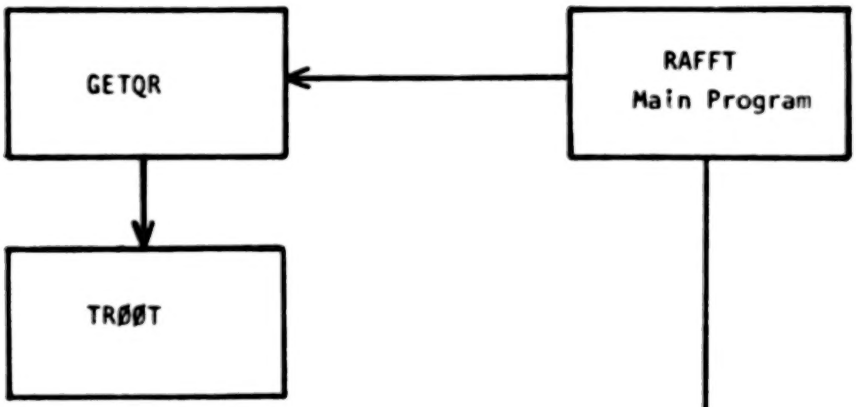
NAME	DESCRIPTION
FFT	Converts a complex array into two real arrays, and then calls FFT2.
FFT2	A fast Fourier transform subroutine written in assembly language*.
GREENS	Calculates the Green's functions H_m, J_m and K_m in accord with Equation (84) for m = 0, 1, 2....NY-1. In this routine the expression $f(W)=W=\sqrt{W^2-1}$ occurs where $W = \Omega$ (Equation (57)). The sign of the square root is chosen in such a way that f(W) is symmetric in the imaginary part and antisymmetric in the real part, with $f(\infty)=+i$. To calculate $H_1(t)$ we evaluate the integral in a finite interval from -WCUT to +WCUT using the fast Fourier transform, where the cut-off frequency is calculated as WCUT=Y/C/DTIME. The contribution from the tails outside the interval (-WCUT,+WCUT) is just 1/2 the delta function, plus terms of second order (i.e., containing the factor 1/WCUT**2).
PØLY	Evaluates a polynomial, using as few multiplications as possible.
TCØNV	Forms the convolution of two arrays A and B, calculating only one new value $$S_i = \sum_{j=1}^{j_{max}} A(j) * B(i-j) \quad \text{(for one index i=IT)}$$ where j_{max}=min(i, MAXCØN), and MAXCØN is the maximum number of points used for convolution.

*Only suitable for use on CDC 6600 computer systems. However, similar FFT routines are generally available for other systems.

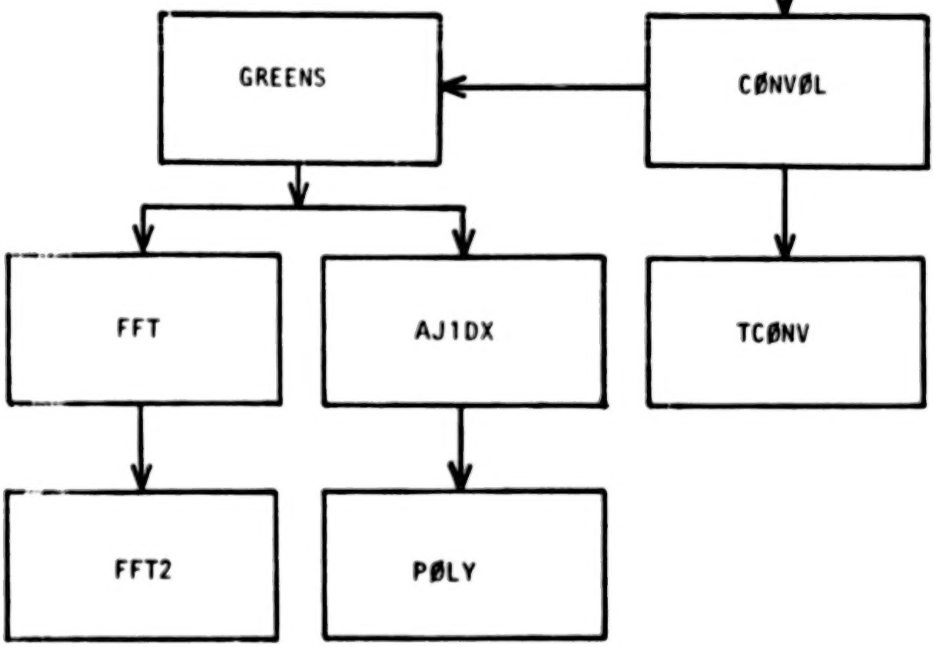
Flow Chart of the Program

The interconnection between the subprograms are represented in the schematic below:

Part 1 (Generates input functions and prints output. Should be replaced by a calling routine from B2DATL.)



Part 2 (Determines acoustic far-field solution)



Test Cases

The input function $Q_n(\tau) = \cos\left(\Omega\,\tau - 2\pi\,\frac{n}{N}\right)\cdot\Delta(\tau)$, where $\tau = 2\pi N a_o t/Y$ is non-dimensional time, and N=NY is the number of points in the y-direction, was used for a test example. In this case $R_n(\tau)$ is the response function, rather than input function.

To check $R_n(\tau)$, it was compared with its asymptotic value. Two cases must be distinguished.

1. If $\Omega \geq n\sqrt{1-M^2}$:

$$R_{n,\,imag}(\tau) = \frac{1}{n(1-M)}\,[\Omega - T_n(\Omega)]\,\sin\Omega\,\tau\,\delta_{n-m} \tag{87}$$

$$R_{n,\,real}(\tau) = \frac{1}{n(1-M)}\,[\Omega - T_n(\Omega)]\,\cos\Omega\,\tau\,\delta_{n-m} \tag{88}$$

$$\text{with}\quad T_n(\Omega) = \sqrt{\Omega^2 - n^2\,(1-M^2)} \tag{89}$$

2. If $\Omega < n\sqrt{1-M^2}$, substitution of $T_n = -\,i|T_n|$ yields:

$$R_{n,\,imag}(\tau) = \frac{1}{n(1-M)}\,[\Omega\,\sin\Omega\,\tau + T_n\,\cos\Omega\,\tau]\,\delta_{n-m} \tag{90}$$

$$R_{n,\,real}(\tau) = \frac{1}{n(1-M)}\,[\Omega\,\cos\Omega\,\tau - T_n\,\sin\Omega\,\tau]\,\delta_{n-m} \tag{91}$$

Two test examples were run, both with m=1, and with $\Omega = 1.0$ in the first case and $\Omega = 0.5$ in the second case. The first test case corresponds to the numerical example discussed in Section IV. Due to the factor δ_{n-m}, $R_n = 0$ except for n=1.

In the first example $\Omega = 1.0 > n\sqrt{1-M^2} = 0.6$. In the second example $\Omega = 0.5 < n\sqrt{1-M^2} = 0.6$.

As shown in Appendix B, R_1 approaches its asymptotic value reasonably well in both cases.

The print-out in Appendix B includes:

- a list of the values of all parameters used
- a comparison of $R(\tau)$ with its asymptotic value
- the input functions Q(IY,IT) and R(IY,IT)
- the Green's functions $H_r(\tau)$ and $J_r(\tau)$ and
- the resulting pressure P(IY,IT).

APPENDIX A

PROGRAM LISTING

```
      PROGRAM WAVE(INPUT,OUTPUT,TAPE98)
      DIMENSION P(8,1001),Q(8,1001),R(8,1001),PU(8,1001)
      LOGICAL FIRST
      DIMENSION ASYMPT(8)
      COMPLEX EPOWER,TROOT,EZPI,W1,W7,E=1,EW7
      COMMON/WSTEP/WSTEP
      EPOWER(X)=CEXP(CMPLX(0.,X))
*
* READ AND PRINT PARAMETERS
 10   READ 110,DTIME,NY,NT,MAXCON,CMARK,RHO0,C,Y,WZERO
 110  FORMAT(F10.6,3I10,4F10.0)
      IF(NT.EQ.0) STOP
      PRINT120,DTIME,NY,NT,MAXCON,CMARK,RHO0,C,Y,WZERO
 120  FORMAT(*1PARAMETERS USED*/
     ** DTIME  **E10.3/
     ** NY     **I10/
     ** NT     **I10/
     ** MAXCON **I10/
     ** CMARK  **E10.3/
     ** RHO0   **E10.3/
     ** C      **E10.3/
     ** Y      **E10.3/
     ** OMEGA0 **E10.3/ )
*
      NSTEP=5
      NDIM=8
      NDIM1=1001
      FIRST=.TRUE.
*
 11   CALL SECOND(T0)
      PRINT 114
 114  FORMAT(///* TEST   RISTAR APPROACHES ITS ASYMPTOTIC VALUE*//)
      DO 12 IT=1,NT
* GET NEW VALUES OF Q AND R AT TIME T FOR NY DISCRETE VALUES OF Y
      CALL        GETQR(Q,R,IT,DTIME,NY,CMARK,C,Y,NDIMY,NDIM1,WZERO)
 12   CONTINUE
*
      PRINT 115
 115  FORMAT(*1       TIME     INPUT P(TIME) *)
      DO 15 IT=1,NT,NSTEP
      TIME=(IT-1)*DTIME
 15   PRINT 125,TIME,(P(IY,IT),IY=1,NY)
 125  FORMAT(1X,F10.3,F9.3,7F7.3)
      PRINT 116
 116  FORMAT(*1       TIME     INPUT Q(TIME)*)
      DO 16 IT=1,NT,NSTEP
      TIME=(IT-1)*DTIME
 16   PRINT 125,TIME,(Q(IY,IT),IY=1,NY)
      CALL SECOND(T1)
      TUSED=T1-T0
      PRINT 310,TUSED
*
      DO 20 IT=1,NT
* CALCULATE P(Y,T) FOR DISCRETE VALUES OF Y
* BY CONVOLUTION WITH GREEN'S FUNCTIONS
      CALL        CONVOL(P,Q,R,NY,IT,DTIME,MAXCON,CMARK,RHO0,C,Y,FIRST,
     *                   NDIMY,NDIM1,PU)
*
* PRINT SELECTED VALUES OF THE RESULTING PRESSURE P
      IF(MOD(IT-1,NSTEP),NE,0) GO TO 20
      IF(IT.EQ.1) PRINT 130
 130  FORMAT(*1       TIME     PRESSURE P(TIME) FOR DISCRETE VALUES OF Y*)
      IF(IT.EQ.1) CALL SECOND(T1)
      TIME=(IT-1)*DTIME
      PRINT 140,TIME, (P(IY,IT),IY=1,NY)
 140  FORMAT(1X,9E10.3)
 20   CONTINUE
      CALL SECOND(T2)
      TUSED=T2-T1
      PRINT 310,TUSED
 310  FORMAT(///* TIME USED **F7.3* SECONDS*//)
      PRINT 150
 150  FORMAT(*1       TIME     UPSTREAM COMPONENT OF PRESSURE*)
      DO 50 IT=1,NT,NSTEP
      TIME=(IT-1)*DTIME
 50   PRINT 140,TIME,(PU(IY,IT),IY=1,NY)
      END
```

```
      SUBROUTINE GETG4(G,R,IT,DTIME,NY,CMARK,C,Y,NDIMY,NDIMT,mZERO)

      DIMENSION G(NDIMY,NDIMT),R(NDIMY,NDIMT)
      REAL J6(1001),J7(1001)
      COMPLEX GISTAR(1001),J7STAR(1001),RISTAR,R7STAR,RR(8)
      COMPLEX ASYMPT,TWOUT
      COMMON/NSTEP/NSTEP
      PI=3.14159265358979
      TWOPI=2.*PI
      T=(IT-1)*DTIME
      ALPHAC=TWOPI*C/Y
      mZALPHAC=mZERO
      MDT=mZ*T
      A=SQRT(1.-CMARK**2)
*
      DO 10 IY=1,NY
      G(IY,1)=COS(MDT-TWOPI*(IY-1)/NY)
 10   CONTINUE
*
      J1(IT)=(1.+CMARK)*AJ1DX(ALPHAC*A*T)
      GISTAR(IT)=.5*CEXP(CMPLX(0.,-MDT))
      RISTAR=0.
      DO 20 JT=1,IT
      RISTAR=RISTAR+J1(JT)*GISTAR(IT-JT+1) *(0.,1.)   /PI
 20   CONTINUE
*
      IF(MOD(IT-1,NSTEP),NE.0) GO TO 25
      ASYMPT= (1./(1.-CMARK)) * (mZERO-TWOUT(1,CMARK,mZERO))
     *    * CEXP(CMPLX(0.,-MDT))
      PRINT 125,IT,RISTAR,ASYMPT
 125  FORMAT(' TIME STEP='I4'   RISTAR ='2F7.3'   ASYMPTOTE ='2F7.3)
 25   CONTINUE
*
      DO 30 IY=1,NY
      R(IY,IT)=REAL(     CEXP(CMPLX(0.,-TWOPI*(IY-1)/NY))*RISTAR   )
 30   CONTINUE
      RETURN
      END
```

```
      COMPLEX FUNCTION TWOUT(IY,CMARK,M)
      S=M**2-IY**2*(1.-CMARK**2)
      IF(S.GE.0.) TWOUT=SQRT(S)
      IF(S.LT.0.) TWOUT=(0.,-1.)*SQRT(-S)
      RETURN
      END
```

```
      SUBROUTINE CONVOL(P,Q,R,NX,IT,DTIME,MAXCON,CMARK,RMDD,C,Y,FIRST,
     *                  NDIMX,NDIMT,PU)
*
* GIVEN THE INPUT FUNCTIONS Q(X,T) = (P*RMDD*C*U)/(RMDD*C) AND R(X,T)=X,
* FIND THE PRESSURE P AT X=D THROUGH CONVOLUTION IN TIME AND SPACE
* P(IX,IT) = RMDD*C*(1/2) * SUM(OVER NX FROM 0 TO NX-1) OF
*            ( Q(NX,IT) CONV H(IX-NX,IT) + R(NX,IT) CONV J(IX-NX,IT) )
*
      DIMENSION P(NDIMX,NDIMT),Q(NDIMX,NDIMT),R(NDIMX,NDIMT)
      DIMENSION QCONV(0,0),RCONV(0,0)
      REAL HH(1001,5),JH(1001,5)
      REAL PU(NDIMX,NDIMT),QRCONV(0,0),AR(1001,5)
      LOGICAL FIRST
      COMMON/PI/PI,PIHALF,TWOPI
      DATA PI/3.14159265358979/
*
* IF THIS SUBROUTINE IS CALLED FOR THE FIRST TIME,
* CALCULATE THE GREENS FUNCTIONS HH AND JH
* AND THE NON-DIMENSIONAL TIME STEP DT
      IF(.NOT.FIRST) GO TO 10
      PIHALF=PI/2.
      TWOPI=2.*PI
      RMDDC=RMDD*C/4.*C
      DT=DTIME*C*TWOPI/Y
      NX2=NX/2
      NX21=NX2+1
      NX22=NX2+2
      NDIMP=NDIMX+NDIMT/2
      CALL SECOND(T0)
      CALL GREENS(HH,JH,DT,MAXCON,NX,CMARK,P,NDIMP,AR)
      CALL SECOND(T1)
      TUSED=T1-T0
      PRINT 310,TUSED
  310 FORMAT(//' TIME USED =',F7.3,' SECONDS'//)
      FIRST=.FALSE.
*
* PREPARE A TABLE OF TIME CONVOLUTIONS
* (SAVES COMPUTER TIME AT USING SYMMETRIES)
   10 DO 50 IX=1,NX
      DO 20 NX=1,NX21
      CALL TCONV(Q(IX,1),HH(1,NX),IT,DT,NX,MAXCON,QCONV(IX,NX))
      CALL TCONV(Q(IX,1),JH(1,NX),IT,DT,NX,MAXCON,QRCONV(IX,NX))
   20 CONTINUE
      DO 30 NX=NX22,NX
      NX2=2*NX+1
      QCONV(IX,NX)=QRCONV(IX,NX2)
   30 QCONV(IX,NX)=QCONV(IX,NX)
*
      RCONV(IX,1)=0.
      RCONV(IX,NX21)=0
      DO 40 NX=2,NX2
      NX1=2*NX-1
      CALL TCONV(R(IX,1),JH(1,NX-1),IT,DT,NX,MAXCON,RCONV(IX,NX))
   40 RCONV(IX,NX)=QRCONV(IX,NX)
   50 CONTINUE
*
      DO 70 IX=1,NX
      SUMQ=0.
      SUMR=0.
      SUMA=0.
      DO 60 NX=1,NX
      IXNX=IX-NX+1
      IF(IXNX.LT.1) IXNX=IXNX+NX
      SUMQ=SUMQ+QCONV(NX,IXNX)
      SUMR=SUMR+RCONV(NX,IXNX)
      SUMA=SUMA+QRCONV(NX,IXNX)
   60 CONTINUE
      PU(IX,IT)=RMDDC*SUMA
   70 P(IX,IT)=RMDDC*(SUMQ+SUMR)
      RETURN
      END
```

```
      SUBROUTINE GREENS(HR,JR,DT,MAXCOS,NT,CMAX,P,NDIMP,KR)
*
* GENERATE THE GREENS FUNCTIONS HR(T) AND JR(T)
* FOR T = 0, DT, 2*DT, ...., (MAXCOS-1)*DT
* AND FOR R = 0, 1, ...., NT/2 (FOR R),  R = 1, 2, ...., NT/2+1 (FOR J)
*
      REAL HR(1001,5),JR(1001,5),COSJR(8),SINJR(8)
      REAL KR(1001,5)
      COMPLEX S,P(1)
      COMMON/PI/PI,PIHALF,TWOPI
*
* P, WHICH IS NOT DEFINED AT THE BEGINNING, IS USED
* AS DUMMY STORAGE FOR THE FAST FOURIER TRANSFORM
*
* FIND MAXIMUM N WHICH FITS WITHIN SPACE OF P
      N=16384
 10   IF(N.LE.NDIMP) GO TO 20
      N=N/2
      GO TO 10
*
 20   CMAX=CMAX**2
      ABSQRT(CM)
      NCUT=ALOG(1./DT
      DAMPCUT=0
      NT2=NT/2
      NT2P1=NT2+1
      NTM1=NT-1
      NT2M1=NT2-1
      NUMN=0
* TABLES OF SINES AND COSINES
      DO 25 I=1,99
      ARG=I*PI*(I-1)/99
      SINJR(I)=SIN(ARG)
 25   COSJR(I)=COS(ARG)
*
* CALCULATE HI
 1000 FORMAT(1X,F10.5,F9.5,F9.7,5)
 1100 FORMAT(*       TIME     GREENS FUNCTION HR*)
      PRINT 1100
      DO 30 I=1,N
      HR(I+1)=0.
      SQ=I**2*CP
      IF(SQ.GE.0.) S=SQRT(SQ)
      IF(SQ.LT.0.) S=CMPLX(0.,-SQRT(-SQ))
 30   P(I)=(N-5)*EXP(S
*
      CALL FFT(P,N)
*
      DO 40 I=1,NQ
* HR IS EVEN, TAKE TWICE REAL PART OF FOURIER TRANSFORM
      P(I)=REAL(P(I))*DT/PI
* DELTA FUNCTION
      IF(I.EQ.1) P(I)=.5/DT
 40   CONTINUE
*
* COSINE SERIES
      DO 70 I=1,MAXCOS
      DO 60 J=1,NT2I
      JEJ1=I
      SUM=0.
* HR = 1/2 * DELTA FUNCTION
      IF(I.EQ.1) SUM=.5/DT
      DO 50 K=1,NTM1
      IF(K*I       .GT.NQ) GO TO 50
      INDEX=MOD(JEK,NT)+1
      IF(I.GT.1) SUM=SUM+(COSJR(INDEX) * KRP(K*I       )
      IF(I.EQ.1) SUM=SUM+(COSJR(INDEX) * P(I)
 50   CONTINUE
 60   HR(I,JI)=SUM
      IF(MOD(I,50).EQ.2 .AND. I.NE.2) PRINT 1100
      T=(I-1)*DT
 100  PRINT 1000,T,(HR(I,J),J=1,NT2I)
 70   CONTINUE
*
* JN (FOR N=1) IS THE BESSEL FUNCTION M = J1(A*T)/(A*T)
* WITH A = SQRT(1+**2)
 1190 FORMAT(*       TIME     GREENS FUNCTION JR*)
      PRINT 1190
      DO 80 I=1,NQ
      T=(I-1)*DT
 80   P(I)=CMARK*AJ1DX(A*T)
```

```
C
C SINE SERIES
      DO 190 I=1,MAXCOL
      DO 100 J=1,NY2M1
      SUM=0.
      DO 90 K=1,NYM1
      IF(K*I       .GT.NQ) GO TO 90
      INDEX=MOD(J*K,NY)+1
      SUM=SUM+SIN(J*(INDEX) * X*P(K*I       )
   90 CONTINUE
  100 JR(I,J)=SUM
      IF(MOD(I,50).EQ.2 .AND. I.NE.2) PRINT 1190
      T=(I-1)*DT
      PRINT 1000,T,(JR(I,J),J=1,NY2M1)
  190 CONTINUE
C
C CALCULATE K1
 1200 FORMAT(*1       TIME    GREENS FUNCTION K**)
      PRINT 1200
      DO 130 I=1,N
      X(I-1)=DX
      S=X**2+CP
      IF(SQ.GE.0.) S=SQRT(SQ)
      IF(SQ.LT.0.) S=CMPLX(0.,+SQRT(+SQ))
  130 P(I)=(X-S)*(X+CMARK*S)/(CMPLX(1.,0.*CMARK))
      CALL FFT(P,N)
      DO 140 I=1,NQ
      P(I)=REAL(P(I))*DX/PI
      IF(I.EQ.1) P(I)=.5/DT
  140 CONTINUE
      DO 170 I=1,MAXCOL
      DO 160 J1=1,NY21
      J=J1-1
      SUM=0.
      IF(I.EQ.1) SUM=.5/DT
      DO 150 K=1,NYM1
      IF(K*I       .GT.NQ) GO TO 150
      INDEX=MOD(J*K,NY)+1
      IF(I.GT.1) SUM=SUM+COSJ*(INDEX)*X*P(K*I       )
      IF(I.EQ.1) SUM=SUM+COSJ*(INDEX)*P(1)
  150 CONTINUE
  160 KR(I,J1)=SUM
      IF(MOD(I,50).EQ.2 .AND. I.NE.2) PRINT 1200
      T=(I-1)*DT
  200 PRINT 1000,T,(KR(I,J),J=1,NY21)
  170 CONTINUE
C
      RETURN
      END
```

```
      SUBROUTINE TCONV(GR,HJ,IT,DT,NY,MAXCON,RESULT)
*
* CONVOLUTION IN TIME
* OF G(IY,IT) WITH H(JY,IT)
* OR H(IY,IT) WITH J(JY,IT)
* TO CALCULATE ONE NEW POINT
*
      DIMENSION GR(1),HJ(1)
      RESULT=0.
      MAXT=MAXCON
      IF(IT.LT.MAXCON) MAXT=IT
      J=(IT-1)*NY+1
      DO 10 K=1,MAXT
      RESULT=RESULT+GR(J)*HJ(K)
 10   J=J-NY
      RESULT=RESULT*DT
      RETURN
      END
```

```
      FUNCTION AJ1DX(X)
* BESSEL FUNCTION J1(X)/X
      DIMENSION C1(7),C2(7),C3(7)
      DATA (C1(I),I=1,7)/.5,-.56249985,.21093573,-.03954289,
     *                   .00443319,-.00031761,.00001109/
      DATA (C2(I),I=1,7)/.79788456,.00000156,.01659667,.00017105,
     *                   .00249511,-.00113653,.00020033/
      DATA (C3(I),I=1,7)/-2.35619449,.12499612,-.00005650,-.00637879,
     *                   .00074348,.00079824,-.00029166/
      AX=ABS(X)
      IF(AX.GT.3.) GO TO 10
      AJ1DX=POLY(C1,(X/3.)**2,7)
      RETURN
 10   F1=POLY(C2,3./AX,7)
      THETA1=AX+POLY(C3,3./AX,7)
      AJ1DX=F1*COS(THETA1)/SQRT(AX)/AX
      RETURN
      END
```

```
      FUNCTION POLY(A,X,N)
* EVALUATE THE POLYNOMIAL A(1)+A(2)*X+...+A(N)*X**(N-1)
      DIMENSION A(1)
      PO,Y=0.
      XPOWER=1.
      DO 10 I=1,N
      POLY=POLY+A(I)*XPOWER
 10   XPOWER=XPOWER*X
      RETURN
      END
```

```
      SUBROUTINE FFT(XREAL,NSIG)
      REAL XREAL(1)
      IF(NSIG.LT.0) GO TO 10
      N=NSIG
      NSPACE=2
      GO TO 20
 10   N=-NSIG
      NSPACE=-2
 20   CALL FFT2(XREAL,XREAL(2),N,NSPACE)
      RETURN
      END
```

```
         IDENT FFT2
         ENTRY FFT2
L100     SX0   B3
         SB4   B0
         SB5   B3-B7
         AX0   1
         SB6   B0
         SB7   X0
         SX1   B5
         EQ    B6,B7,FFT23
L110     SB4   B3-B4
         SB5    B3-B5
         SA2   B1+B4
         SA3   B1+B5
         SA4   B2+B4
         AX7    X2
         SA5   B2+B5
         AX6   X3
         SA7   A3
         SA6   A2
         AX7   X4
         AX6   X5
         SA7   A5
         SA6   A4
         LT    B6,B4,L110
L120     SB4   B4+B7
         SB5   B4+B5
         SA2   B1+B4
         SA3   B1+B5
         SA4   B2+B4
         AX7   X2
         SA5   B2+B5
         AX6   X3
         SA7   A3
         SA6   A2
         AX7   X4
         SX0   B6
         AX6   X5
         SA7   A5
         SA6   A4
L130     AX0   1
         IX1   X1+X0
         PL    X1,L130
         LX0   1
         SB4   B4+B7
         IX1   X1+X0
         SB5   X1
         GE    B5,B4,L110
         LT    B4,B6,L120
FFT23    SA1   A0
FFT2
         SA0    A1
         SA1  A0
         SB1  X1
         SA1  A0+1
         SB2  X1
         SA1  A0+2
         SB3  X1
         SA1  A0+3
         SB4  X1
         SA0     B4
         MX2      1
         SA5     L60
         SA3     B3
         LX2     57
         PX7     X3
         RX6     -X2*X5
          PL     X0,L10
         RX0     X2+X5
         RX4     -X4
L10      LX3     32
         SA6     A5
         MX0     B5,X3
         PX2     X4
         SB7     X4
         DX7     X2*X7
         SA1     B5+5
         SB5     X7
         SA6     X7
         EQ      L40
L20      SA3     C0
         PX4     X2+X1
         PX7     X2+X0
         RX5     X3-X0
         RX6     X3+X1
         RX4     X4-X5
         RX6     X6+X7
         MX5     X4
         PX7      X1-X6
         RX0     X0+X5
         MX1     X7
L30      SB5     B6+B4
         SA2     B1+B4
         SA3     B1+B5
         SA4     B2+B4
         RX6     X2+X3
         SA5     B2+B5
         PX2     X2-X3
         SA6     A2
         PX7     X4+X5
         PX3     X1+X2
         PX4     X4-X5
         SA7     A4
         PX5     X0+X4
         PX2     X6+X2
         PX6     X3-X5
         PX4     X1+X4
         SA6     A3
         PX7     X2+X4
         SB4     B6+B5
         SA7     A5
         LT      B4,B3,L30
         SB5     B4-B3
         MX1     -X1
         SB4     B6-B5
         LT      B5,B4,L30
         SB4     B4+B7
         SA2     B0
         LT      B4,B5,L20
L40      SB4     B0
         SX5     B6
         AX5     1
         SB6     X5
L50      SB5     B6+B4
         SA2     B1+B4
         SA3     B1+B5
         SA4     B2+B4
         RX6     X2+X3
         SA5     B2+B5
         RX7     X2-X3
         SA6     A2
         SA7     A3
         RX6     X4+X5
         SB4     B6+B5
         RX7     X4-X5
         SA6     A4
         SA7     A5
         LT      B4,B3,L50
         EQ      B6,B7,L100
         SA1     A1
         SB4     B7
         RX6     X1+X1
         SA1     A1+1
         PX6     X6+X6
          SA3    ONE
          SA6    C0
L60      MX0     X1
         MX6     X3-X6
         PX7     X0
         MX1     X6
         SA7     B0
         EQ      L30
S        DATA    9.5873709095977306E-5
         DATA    1.9174759731070331E-4
         DATA    3.8349518757139550E-4
         DATA    7.6699831675225053E-4
         DATA    1.5334801808214868E-3
         DATA    3.0679567629659763E-3
         DATA    6.1358846491544753E-3
         DATA    1.2271538285719922E-2
         DATA    2.4541228522912288E-2
         DATA    4.9067674327418015E-2
         DATA    9.8017140329560601E-2
         DATA    1.9509032201612827E-1
         DATA    3.8268343236508977E-1
         DATA    0.7071067811865475
ONE      DATA    1.0
C0
B0
         L40
```

APPENDIX B

PRINTOUT OF TEST CASES

```
PARAMETERS USED
DTIME  =  .250E-05
NY     =          8
NT     =        201
MAXCON =        101
CMARK  =  .800E+00
RHO0   =  .100E+01
C      =  .100E+04
Y      =  .100E+00
OMEGA0 =  .100E+01


TEST   RISTAR APPROACHES ITS ASYMPTOTIC VALUE

TIME STEP   1   RISTAR =  0.000   .143   ASYMPTOTE =  1.000  0.000
TIME STEP   6   RISTAR =  -.316   .757   ASYMPTOTE =   .707   .707
TIME STEP  11   RISTAR =  -.963   .928   ASYMPTOTE =   .000  1.000
TIME STEP  16   RISTAR = -1.520   .520   ASYMPTOTE =  -.707   .707
TIME STEP  21   RISTAR = -1.598  -.257   ASYMPTOTE = -1.000   .000
TIME STEP  26   RISTAR = -1.065  -.969   ASYMPTOTE =  -.707  -.707
TIME STEP  31   RISTAR =  -.150 -1.209   ASYMPTOTE =  -.000 -1.000
TIME STEP  36   RISTAR =   .701  -.835   ASYMPTOTE =   .707  -.707
TIME STEP  41   RISTAR =  1.068  -.057   ASYMPTOTE =  1.000  -.000
TIME STEP  46   RISTAR =   .801   .687   ASYMPTOTE =   .707   .707
TIME STEP  51   RISTAR =   .101   .983   ASYMPTOTE =   .000  1.000
TIME STEP  56   RISTAR =  -.596   .680   ASYMPTOTE =  -.707   .707
TIME STEP  61   RISTAR =  -.874  -.023   ASYMPTOTE = -1.000   .000
TIME STEP  66   RISTAR =  -.580  -.698   ASYMPTOTE =  -.707  -.707
TIME STEP  71   RISTAR =   .098  -.980   ASYMPTOTE =  -.000 -1.000
TIME STEP  76   RISTAR =   .738  -.603   ASYMPTOTE =   .707  -.707
TIME STEP  81   RISTAR =   .988   .113   ASYMPTOTE =  1.000  -.000
TIME STEP  86   RISTAR =   .577   .782   ASYMPTOTE =   .707   .707
TIME STEP  91   RISTAR =  -.159  1.004   ASYMPTOTE =   .000  1.000
TIME STEP  96   RISTAR =  -.835   .638   ASYMPTOTE =  -.707   .707
TIME STEP 101   RISTAR = -1.052  -.110   ASYMPTOTE = -1.000   .000
TIME STEP 106   RISTAR =  -.672  -.806   ASYMPTOTE =  -.707  -.707
TIME STEP 111   RISTAR =   .094 -1.046   ASYMPTOTE =  -.000 -1.000
TIME STEP 116   RISTAR =   .810  -.687   ASYMPTOTE =   .707  -.707
TIME STEP 121   RISTAR =  1.066   .063   ASYMPTOTE =  1.000  -.000
TIME STEP 126   RISTAR =   .717   .772   ASYMPTOTE =   .707   .707
TIME STEP 131   RISTAR =  -.031  1.029   ASYMPTOTE =   .000  1.000
TIME STEP 136   RISTAR =  -.743   .669   ASYMPTOTE =  -.707   .707
TIME STEP 141   RISTAR = -1.010  -.045   ASYMPTOTE = -1.000   .000
TIME STEP 146   RISTAR =  -.693  -.743   ASYMPTOTE =  -.707  -.707
TIME STEP 151   RISTAR =   .039  -.996   ASYMPTOTE =  -.000 -1.000
TIME STEP 156   RISTAR =   .726  -.660   ASYMPTOTE =   .707  -.707
TIME STEP 161   RISTAR =   .974   .065   ASYMPTOTE =  1.000  -.000
TIME STEP 166   RISTAR =   .637   .751   ASYMPTOTE =   .707   .707
TIME STEP 171   RISTAR =  -.084   .991   ASYMPTOTE =   .000  1.000
TIME STEP 176   RISTAR =  -.762   .684   ASYMPTOTE =  -.707   .707
TIME STEP 181   RISTAR =  -.993  -.088   ASYMPTOTE = -1.000   .000
TIME STEP 186   RISTAR =  -.637  -.775   ASYMPTOTE =  -.707  -.707
TIME STEP 191   RISTAR =   .102 -1.011   ASYMPTOTE =  -.000 -1.000
TIME STEP 196   RISTAR =   .791  -.657   ASYMPTOTE =   .707  -.707
TIME STEP 201   RISTAR =  1.028   .085   ASYMPTOTE =  1.000  -.000
```

TIME	INPUT R(TIME)							
0.	0.000	.101	.143	.101	-.000	-.101	-.143	-.101
.125E-04	-.316	.312	.757	.759	.316	-.312	-.757	-.759
.250E-04	-.963	-.025	.928	1.337	.963	.025	-.928	-1.337
.375E-04	-1.520	-.707	.520	1.442	1.520	.707	-.520	-1.442
.500E-04	-1.596	-1.310	-.257	.946	1.596	1.310	.257	-.946
.625E-04	-1.065	-1.439	-.969	.068	1.065	1.439	.969	-.068
.750E-04	-.150	-.961	-1.209	-.749	.150	.961	1.209	.749
.875E-04	.701	-.095	-.835	-1.066	-.701	.095	.835	1.066
.100E-03	1.068	.715	-.057	-.796	-1.068	-.715	.057	.796
.113E-03	.801	1.052	.687	-.081	-.801	-1.052	-.687	.081
.125E-03	.101	.766	.983	.623	-.101	-.766	-.983	-.623
.138E-03	-.596	.060	.580	.902	.596	-.060	-.580	-.902
.150E-03	-.874	-.634	-.023	.602	.874	.634	.023	-.602
.163E-03	-.580	-.904	-.698	-.084	.580	.904	.698	.084
.175E-03	.098	-.598	-.940	-.734	-.098	.598	.940	.734
.188E-03	.738	.095	-.603	-.948	-.738	-.095	.603	.948
.200E-03	.944	.748	.113	-.588	-.944	-.748	-.113	.588
.213E-03	.577	.962	.782	.145	-.577	-.962	-.782	-.145
.225E-03	-.159	.598	1.004	.822	.159	-.598	-1.004	-.822
.238E-03	-.835	-.140	.638	1.041	.835	.140	-.638	-1.041
.250E-03	-1.052	-.822	-.110	.666	1.052	.822	.110	-.666
.263E-03	-.672	-1.046	-.806	-.095	.672	1.046	.806	.095
.275E-03	.094	-.673	-1.046	-.806	-.094	.673	1.046	.806
.288E-03	.810	.096	-.687	-1.058	-.810	-.096	.687	1.058
.300E-03	1.066	.798	.063	-.709	-1.066	-.798	-.063	.709
.313E-03	.717	1.053	.772	.039	-.717	-1.053	-.772	-.039
.325E-03	-.031	.706	1.029	.749	.031	-.706	-1.029	-.749
.338E-03	-.743	-.038	.689	1.013	.743	.038	-.689	-1.013
.350E-03	-1.010	-.746	-.085	.683	1.010	.746	.085	-.683
.363E-03	-.683	-1.008	-.743	-.042	.683	1.008	.743	.042
.375E-03	.039	-.677	-.996	-.732	-.039	.677	.996	.732
.388E-03	.726	.047	-.660	-.980	-.726	-.047	.660	.980
.400E-03	.974	.735	.065	-.643	-.974	-.735	-.065	.643
.413E-03	.637	.981	.751	.080	-.637	-.981	-.751	-.080
.425E-03	-.084	.642	.991	.760	.084	-.642	-.991	-.760
.438E-03	-.762	-.083	.644	.994	.762	.083	-.644	-.994
.450E-03	-.993	-.764	-.088	.640	.993	.764	.088	-.640
.463E-03	-.637	-.998	-.775	-.097	.637	.998	.775	.097
.475E-03	.102	-.643	-1.011	-.787	-.102	.643	1.011	.787
.488E-03	.791	.095	-.657	-1.024	-.791	-.095	.657	1.024
.500E-03	1.028	.787	.085	-.667	-1.028	-.787	-.085	.667

TIME	INPUT D(TIME)							
0.	1.000	.707	-.000	-.707	-1.000	-.707	.000	.707
.125E-04	.707	1.000	.707	.000	-.707	-1.000	-.707	-.000
.250E-04	.000	.707	1.000	.707	-.000	-.707	-1.000	-.707
.375E-04	-.707	.000	.707	1.000	.707	-.000	-.707	-1.000
.500E-04	-1.000	-.707	.000	.707	1.000	.707	.000	-.707
.625E-04	-.707	-1.000	-.707	.000	.707	1.000	.707	.000
.750E-04	-.000	-.707	-1.000	-.707	.000	.707	1.000	.707
.875E-04	.707	-.000	-.707	-1.000	-.707	.000	.707	1.000
.100E-03	1.000	.707	-.000	-.707	-1.000	-.707	.000	.707
.113E-03	.707	1.000	.707	-.000	-.707	-1.000	-.707	.000
.125E-03	.000	.707	1.000	.707	-.000	-.707	-1.000	-.707
.138E-03	-.707	-.000	.707	1.000	.707	-.000	-.707	-1.000
.150E-03	-1.000	-.707	-.000	.707	1.000	.707	-.000	-.707
.163E-03	-.707	-1.000	-.707	-.000	.707	1.000	.707	-.000
.175E-03	-.000	-.707	-1.000	-.707	-.000	.707	1.000	.707
.188E-03	.707	-.000	-.707	-1.000	-.707	.000	.707	1.000
.200E-03	1.000	.707	-.000	-.707	-1.000	-.707	.000	.707
.213E-03	.707	1.000	.707	-.000	-.707	-1.000	-.707	.000
.225E-03	.000	.707	1.000	.707	-.000	-.707	-1.000	-.707
.238E-03	-.707	.000	.707	1.000	.707	-.000	-.707	-1.000
.250E-03	-1.000	-.707	-.000	.707	1.000	.707	-.000	-.707
.263E-03	-.707	-1.000	-.707	-.000	.707	1.000	.707	-.000
.275E-03	-.000	-.707	-1.000	-.707	-.000	.707	1.000	.707
.288E-03	.707	-.000	-.707	-1.000	-.707	.000	.707	1.000
.300E-03	1.000	.707	-.000	-.707	-1.000	-.707	.000	.707
.313E-03	.707	1.000	.707	-.000	-.707	-1.000	-.707	.000
.325E-03	.000	.707	1.000	.707	-.000	-.707	-1.000	-.707
.338E-03	-.707	.000	.707	1.000	.707	-.000	-.707	-1.000
.350E-03	-1.000	-.707	-.000	.707	1.000	.707	-.000	-.707
.363E-03	-.707	-1.000	-.707	-.000	.707	1.000	.707	-.000
.375E-03	-.000	-.707	-1.000	-.707	-.000	.707	1.000	.707
.388E-03	.707	-.000	-.707	-1.000	-.707	.000	.707	1.000
.400E-03	1.000	.707	-.000	-.707	-1.000	-.707	.000	.707
.413E-03	.707	1.000	.707	-.000	-.707	-1.000	-.707	.000
.425E-03	.000	.707	1.000	.707	-.000	-.707	-1.000	-.707
.438E-03	-.707	.000	.707	1.000	.707	-.000	-.707	-1.000
.450E-03	-1.000	-.707	-.000	.707	1.000	.707	-.000	-.707
.463E-03	-.707	-1.000	-.707	-.000	.707	1.000	.707	-.000
.475E-03	-.000	-.707	-1.000	-.707	-.000	.707	1.000	.707
.488E-03	.707	-.000	-.707	-1.000	-.707	.000	.707	1.000
.500E-03	1.000	.707	-.000	-.707	-1.000	-.707	.000	.707

TIME	GREENS FUNCTION HR				
0.000	25.405	-.000	-.000	-.000	0.000
.157	-1.681	.078	.284	.317	.324
.314	-2.305	.183	.370	.398	.404
.471	-2.630	.332	.389	.395	.397
.628	-2.688	.508	.341	.318	.314
.785	-2.408	.678	.240	.194	.185
.942	-1.999	.800	.111	.060	.050
1.100	-1.525	.855	-.016	-.049	-.055
1.257	-1.081	.815	-.113	-.100	-.107
1.414	-.731	.688	-.159	-.111	-.104
1.571	-.505	.497	-.148	-.089	-.058
1.728	-.395	.280	-.077	-.007	.003
1.885	-.367	.078	.030	.051	.054
2.042	-.379	-.083	.153	.082	.076
2.199	-.392	-.180	.267	.077	.064
2.356	-.384	-.213	.351	.040	.028
2.513	-.348	-.197	.390	-.011	-.016
2.670	-.293	-.155	.382	-.056	-.047
2.827	-.236	-.110	.332	-.078	-.056
2.985	-.191	-.078	.255	-.064	-.034
3.142	-.167	-.067	.167	-.018	.003
3.299	-.162	-.075	.085	.052	.039
3.456	-.170	-.091	.020	.126	.059
3.613	-.179	-.106	-.020	.189	.053
3.770	-.181	-.111	-.037	.228	.021
3.927	-.172	-.103	-.036	.237	-.023
4.084	-.153	-.087	-.025	.219	-.061
4.241	-.132	-.067	-.013	.183	-.075
4.398	-.114	-.051	-.006	.139	-.051
4.555	-.104	-.044	-.006	.097	.011
4.712	-.104	-.045	-.010	.061	.100
4.869	-.110	-.052	-.020	.034	.197
5.027	-.117	-.060	-.037	.015	.280
5.184	-.120	-.064	-.043	.002	.329
5.341	-.116	-.060	-.043	-.007	.336
5.498	-.106	-.051	-.036	-.011	.303
5.655	-.093	-.040	-.027	-.008	.242
5.812	-.083	-.030	-.017	.004	.170
5.969	-.078	-.025	-.012	.024	.104
6.126	-.079	-.026	-.012	.030	.056
6.283	-.085	-.032	-.018	.027	.031
6.440	-.091	-.038	-.028	.019	.025
6.597	-.096	-.042	-.037	.011	.030
6.754	-.092	-.041	-.042	.010	.037
6.912	-.084	-.036	-.039	.005	.039
7.069	-.075	-.025	-.027	.002	.034
7.226	-.067	-.016	-.008	.006	.022
7.383	-.064	-.011	.017	.022	.008
7.540	-.066	-.013	.041	.026	-.004
7.697	-.072	-.020	.061	-.001	-.010
7.854	-.079	-.029	.073	-.000	-.009

TIME	GWEENS FUNCTION HR				
8.011	-.083	-.036	.075	.004	-.005
8.168	-.081	-.037	.069	.009	.000
8.325	-.074	-.030	.055	.010	.003
8.482	-.064	-.019	.038	.006	.002
8.639	-.055	.006	.021	.002	-.003
8.796	-.052	.027	.009	-.005	-.009
8.954	-.057	.045	.001	-.011	-.014
9.111	-.067	.056	-.001	-.013	-.016
9.268	-.079	.059	.000	-.012	-.015
9.425	-.087	.055	.003	-.008	-.011
9.582	-.085	.046	.005	-.005	-.008
9.739	-.071	.035	.006	-.003	-.005
9.896	-.044	.024	.004	-.003	-.005
10.053	-.009	.015	-.000	-.006	-.008
10.210	.025	.009	-.005	-.010	-.012
10.367	.053	.003	-.008	-.014	-.015
10.524	.067	-.000	-.010	-.015	-.016
10.681	.066	-.003	-.010	-.014	-.015
10.838	.052	-.003	-.007	-.010	-.012
10.996	.029	.000	-.004	-.007	-.008
11.153	.005	.002	-.002	-.005	-.006
11.310	-.015	.017	-.001	-.005	-.006
11.467	-.011	.041	-.003	-.020	-.026
11.624	-.021	.048	-.007	-.019	-.024
11.781	-.026	.052	-.011	-.018	-.020
11.938	-.026	.053	-.015	-.016	-.017
12.095	-.024	.050	-.015	-.015	-.016
12.252	-.021	.045	-.012	-.014	-.016
12.409	-.020	.038	-.005	-.014	-.018
12.566	-.021	.031	.005	-.015	-.021
12.723	-.024	.024	.016	-.017	-.024
12.881	-.029	.019	.027	-.019	-.025
13.038	-.034	.015	.034	-.020	-.025
13.195	-.037	.013	.038	-.021	-.023
13.352	-.023	.013	.021	-.020	-.004
13.509	-.027	.013	.020	-.018	-.003
13.666	-.030	.013	.017	-.013	-.005
13.823	-.032	.013	.013	-.007	-.008
13.980	-.033	.013	.008	.001	-.012
14.137	-.032	.012	.003	.008	-.016
14.294	-.031	.011	-.001	.015	-.019
14.451	-.029	.009	-.005	.020	-.020
14.608	-.028	.008	-.008	.022	-.018
14.765	-.027	.007	-.009	.022	-.012
14.923	-.027	.006	-.010	.019	-.003
15.080	-.028	.006	-.009	.014	.007
15.237	-.030	.006	-.008	.007	.020
15.394	-.032	.007	-.007	.001	.031
15.551	-.034	.007	-.006	-.005	.042
15.708	-.035	.007	-.006	-.009	.049

TIME	GREENS FUNCTION JR		
0.000	-3.698	-1.520	-.629
.157	-3.114	-1.248	-.514
.314	-2.205	-.851	-.348
.471	-1.138	-.420	-.171
.628	-.078	-.058	-.024
.785	.820	.185	.072
.942	1.446	.264	.098
1.100	1.764	.198	.073
1.257	1.788	.050	.020
1.414	1.590	-.108	-.032
1.571	1.267	-.208	-.059
1.728	.912	-.203	-.055
1.885	.601	-.090	-.024
2.042	.375	.105	.017
2.199	.242	.329	.046
2.356	.184	.525	.050
2.513	.172	.650	.024
2.670	.174	.681	-.020
2.827	.170	.625	-.062
2.985	.152	.510	-.080
3.142	.123	.373	-.056
3.299	.091	.249	.012
3.456	.065	.163	.114
3.613	.051	.121	.228
3.770	.048	.113	.325
3.927	.052	.122	.383
4.084	.055	.131	.390
4.241	.054	.128	.348
4.398	.048	.111	.273
4.555	.037	.085	.188
4.712	.027	.059	.117
4.869	.020	.043	.075
5.027	.019	.039	.066
5.184	.021	.046	.082
5.341	.025	.055	.106
5.498	.027	.061	.125
5.655	.026	.058	.120
5.812	.021	.045	.089
5.969	.014	.029	.036
6.126	.009	.010	-.027
6.283	.007	.011	-.086
6.440	.009	.017	-.127
6.597	.013	.031	-.145
6.754	.018	.045	-.139
6.912	.019	.051	-.116
7.069	.015	.042	-.086
7.226	.009	.016	-.057
7.383	.003	-.023	-.037
7.540	-.001	-.066	-.028
7.697	.001	-.103	-.027
7.854	.008	-.125	-.030

TIME	GREENS FUNCTION JR		
8.011	.018	-.129	-.032
8.168	.024	-.116	-.033
8.325	.023	-.093	-.029
8.482	.011	-.066	-.023
8.639	-.013	-.044	-.017
8.796	-.045	-.032	-.013
8.954	-.077	-.029	-.012
9.111	-.101	-.032	-.014
9.268	-.113	-.037	-.016
9.425	-.109	-.039	-.016
9.582	-.093	-.037	-.016
9.739	-.069	-.030	-.013
9.896	-.046	-.022	-.010
10.053	-.029	-.014	-.008
10.210	-.023	-.010	-.007
10.367	-.027	-.010	-.007
10.524	-.037	-.020	-.009
10.681	-.046	-.023	-.010
10.838	-.049	-.024	-.011
10.996	-.041	-.020	-.009
11.153	-.024	-.015	-.007
11.310	.000	-.009	-.005
11.467	.019	-.015	-.010
11.624	.037	-.020	-.013
11.781	.050	-.023	-.013
11.938	.057	-.023	-.011
12.095	.056	-.020	-.008
12.252	.048	-.012	-.001
12.409	.037	-.002	.002
12.566	.025	.011	.003
12.723	.015	.024	.001
12.881	.008	.036	-.004
13.038	.006	.045	-.010
13.195	.007	.050	-.015
13.352	.008	.051	-.010
13.509	.004	.048	-.003
13.666	.005	.042	.006
13.823	.008	.034	.016
13.980	.011	.026	.025
14.137	.013	.019	.033
14.294	.014	.014	.039
14.451	.013	.011	.042
14.608	.011	.011	.042
14.765	.008	.012	.041
14.923	.005	.014	.039
15.080	.003	.016	.035
15.237	.001	.017	.032
15.394	.001	.016	.028
15.551	.001	.015	.025
15.708	.003	.012	.022

TIME	GREENS FUNCTION RR				
0.000	25.465	-.000	-.000	-.000	0.000
.157	-.187	.009	.032	.035	.036
.314	-.256	.020	.041	.044	.045
.471	-.292	.037	.043	.044	.044
.628	-.294	.056	.038	.035	.035
.785	-.268	.075	.027	.022	.021
.942	-.222	.089	.012	.007	.006
1.100	-.169	.095	-.002	-.005	-.006
1.257	-.120	.091	-.013	-.012	-.012
1.414	-.081	.076	-.018	-.012	-.012
1.571	-.056	.055	-.016	-.008	-.006
1.728	-.044	.031	-.009	-.001	.000
1.885	-.041	.008	.003	.006	.006
2.042	-.042	-.009	.017	.009	.008
2.199	-.044	-.020	.030	.009	.007
2.356	-.043	-.024	.039	.006	.003
2.513	-.039	-.022	.043	-.001	-.002
2.670	-.033	-.017	.042	-.006	-.005
2.827	-.026	-.012	.037	-.009	-.006
2.985	-.021	-.009	.028	-.007	-.006
3.142	-.019	-.007	.019	-.002	.000
3.299	-.018	-.008	.009	.006	.006
3.456	-.019	-.010	.002	.014	.007
3.613	-.020	-.012	-.002	.021	.008
3.770	-.020	-.012	-.004	.025	.002
3.927	-.019	-.011	-.004	.026	-.003
4.084	-.017	-.010	-.003	.024	-.007
4.241	-.015	-.007	-.001	.020	-.008
4.398	-.013	-.006	-.001	.015	-.006
4.555	-.012	-.005	-.001	.011	-.001
4.712	-.012	-.005	-.002	.007	.011
4.869	-.012	-.006	-.003	.004	.022
5.027	-.013	-.007	-.004	.002	.031
5.184	-.013	-.007	-.005	.000	.037
5.341	-.013	-.007	-.005	-.001	.037
5.498	-.012	-.006	-.004	-.001	.034
5.655	-.010	-.006	-.003	-.001	.027
5.812	-.009	-.005	-.002	.000	.019
5.969	-.009	-.003	-.001	.003	.012
6.126	-.009	-.003	-.001	.006	.006
6.283	-.009	-.004	-.002	.009	.003
6.440	-.010	-.004	-.003	.011	.003
6.597	-.010	-.005	-.004	.012	.003
6.754	-.010	-.005	-.005	.012	.004
6.912	-.009	-.006	-.006	.011	.004
7.069	-.008	-.006	-.006	.009	.004
7.226	-.007	-.006	-.006	.005	.002
7.383	-.007	-.006	.002	.002	.001
7.540	-.007	-.006	.005	.001	-.000
7.697	-.008	-.002	.007	-.000	-.001
7.854	-.009	-.003	.008	-.000	-.001

TIME	GREENS FUNCTION RR				
8.011	-.009	-.004	.008	.000	-.001
8.168	-.009	-.004	.008	.001	.000
8.325	-.008	-.003	.006	.001	.000
8.482	-.007	-.002	.004	.001	.000
8.639	-.006	.001	.002	.000	-.000
8.796	-.006	.003	.001	-.001	-.001
8.954	-.006	.005	.000	-.001	-.002
9.111	-.007	.006	-.000	-.001	-.002
9.268	-.009	.007	.000	-.001	-.002
9.425	-.010	.006	.000	-.001	-.001
9.582	-.009	.005	.001	-.001	-.001
9.739	-.008	.004	.001	-.000	-.001
9.896	-.005	.003	.000	-.000	-.001
10.053	-.001	.002	-.000	-.001	-.001
10.210	.003	.001	-.001	-.001	-.001
10.367	.006	.000	-.001	-.002	-.002
10.524	.007	-.000	-.001	-.002	-.002
10.681	.007	-.000	-.001	-.002	-.002
10.838	.006	-.000	-.001	-.001	-.001
10.996	.003	.000	-.000	-.001	-.001
11.153	.001	.001	-.000	-.001	-.001
11.310	-.002	.002	-.000	-.001	-.001
11.467	-.001	.005	-.000	-.002	-.003
11.624	-.002	.005	-.001	-.002	-.002
11.781	-.003	.004	-.001	-.002	-.002
11.938	-.003	.006	-.002	-.002	-.002
12.095	-.003	.006	-.002	-.002	-.002
12.252	-.002	.005	-.001	-.002	-.002
12.409	-.002	.004	-.001	-.002	-.002
12.566	-.002	.003	.001	-.002	-.002
12.723	-.003	.003	.002	-.002	-.003
12.881	-.003	.002	.003	-.002	-.003
13.038	-.004	.002	.004	-.002	-.003
13.195	-.004	.001	.004	-.002	-.003
13.352	-.003	.001	.002	-.002	-.000
13.509	-.003	.001	.002	-.002	-.000
13.666	-.003	.001	.002	-.001	-.001
13.823	-.004	.001	.001	-.001	-.001
13.980	-.004	.001	.001	.000	-.001
14.137	-.004	.001	.000	.001	-.002
14.294	-.003	.001	-.000	.002	-.002
14.451	-.003	.001	-.001	.002	-.002
14.608	-.003	.001	-.001	.002	-.002
14.765	-.003	.001	-.001	.002	-.001
14.923	-.003	.001	-.001	.002	-.000
15.080	-.003	.001	-.001	.002	.001
15.237	-.003	.001	-.001	.001	.002
15.394	-.004	.001	-.001	.000	.003
15.551	-.004	.001	-.001	-.001	.005
15.708	-.004	.001	-.001	-.001	.005

TIME	PRESSURE P(TIME) FOR DISCRETE VALUES OF r							
0.	.52bE+0b	.372E+0b	-.7b0E-0B	-.372E+0b	-.52bE+0b	-.372E+0b	-.822E-0B	.372E+0b
.12bE-0b	.b27E+0b	.570E+0b	.379E+0b	-.339E+05	-.b27E+0b	-.570E+0b	-.379E+0b	.339E+05
.25bE-0b	.221E+0b	.b03E+0b	.b31E+0b	.29bE+0b	-.221E+0b	-.b03E+0b	-.b31E+0b	-.29bE+0b
.37bE-0b	-.2b4E+0b	.25bE+0b	.b2bE+0b	.b2bE+0b	.2b4E+0b	-.25bE+0b	-.b2bE+0b	-.b2bE+0b
.50bE-0b	-.b92E+0b	-.321E+0b	.238E+0b	.b5bE+0b	.b92E+0b	.321E+0b	-.238E+0b	-.b5bE+0b
.b2bE-0b	-.b23E+0b	-.b43E+0b	-.3b6E+0b	.322E+0b	.b23E+0b	.b43E+0b	.3b6E+0b	-.322E+0b
.75bE-0b	-.530E+0b	-.102E+07	-.909E+0b	-.2b8E+0b	.530E+0b	.102E+07	.909E+0b	.2b8E+0b
.b75E-0b	.29bE+05	-.752E+0b	-.104E+07	-.794E+0b	.29bE+05	.752E+0b	.104E+07	.794E+0b
.100E-0B	.57bE+0b	-.17bE+0b	-.825E+0b	-.991E+0b	-.57bE+0b	.17bE+0b	.825E+0b	.991E+0b
.113E-0B	.b02E+0b	.397E+0b	-.241E+00	-.737E+0b	-.b02E+0b	-.397E+0b	.241E+0b	.737E+0b
.125E-0B	.b09E+0b	.b79E+0b	.351E+0b	-.142E+0b	-.b09E+0b	-.b79E+0b	-.351E+0b	.1b2E+0b
.13bE-0B	.104E+0b	.537E+0b	.b51E+0b	.383E+0b	-.104E+0b	-.537E+0b	-.b51E+0b	-.383E+0b
.150E-0B	-.393E+0b	.971E+05	.530E+0b	.b53E+0b	.393E+0b	-.971E+05	-.530E+0b	-.b53E+0b
.163E-0B	-.b2bE+0b	-.3b5E+0b	.175E+0b	.513E+0b	.b20E+0b	.3b5E+0b	-.105E+0b	-.513E+0b
.175E-0B	-.---E+0b	-.559E+0b	-.3--E+0b	.721E+05	.---E+0b	.559E+0b	.3--E+0b	-.721E+05
.188E-0B	.719E+04	-.373E+0b	-.53bE+0b	-.383E+0b	-.719E+04	.373E+0b	.53bE+0b	.383E+0b
.200E-0B	.---E+0b	.797E+05	-.347E+0b	-.570E+0b	-.---E+0b	-.797E+05	.347E+0b	.570E+0b
.213E-0B	.b2-E+0b	.515E+0b	.100E+0b	-.373E+0b	-.b28E+0b	-.515E+0b	-.100E+0b	.373E+0b
.225E-0B	.---E+0b	.-59E+0b	.527E+0b	.---E+05	-.---E+0b	-.-59E+0b	-.527E+0b	-.---E+05
.238E-0B	-.---E+05	.---E+0b	.---E+0b	.527E+0b	.---E+05	-.---E+0b	-.---E+0b	-.527E+0b
.25bE-0B	-.551E+0b	-.105E+0b	-.---2E+0b	.---3E+0b	.551E+0b	.105E+0b	.---2E+0b	-.---3E+0b
.263E-0B	-.70bE+0b	-.5b1E+0b	-.115E+0b	-.---1E+0b	.70bE+0b	.5b1E+0b	.115E+0b	-.---1E+0b
.275E-0B	-.---7E+0b	-.747E+0b	-.593E+0b	-.---3E+05	-.---7E+0b	.747E+0b	.593E+0b	.---3E+05
.288E-0B	.57bE+05	-.---3E+0b	-.755E+0b	-.575E+0b	-.57bE+05	.---3E+0b	.755E+0b	.575E+0b
.300E-0B	.5--E+0b	.2bBE+05	-.50-E+0b	-.740E+0b	-.5--E+0b	-.2bBE+05	.50-E+0b	.740E+0b
.313E-0B	.71BE+0b	.521E+0b	.1BBE+05	-.---E+0b	-.71BE+0b	-.521E+0b	-.1BBE+05	.---E+0b
.325E-0B	.---5E+0b	.7B8E+0b	.517E+0b	.22BE+05	-.---5E+0b	-.7B5E+0b	-.517E+0b	-.22BE+05
.338E-0B	-.2B5E+05	.---E+0b	.7BBE+0b	.515E+0b	.2B5E+05	-.---E+0b	-.7BBE+0b	-.515E+0b
.350E-0B	-.53-E+0b	-.---E+04	.---E+0b	.702E+0b	.53-E+0b	.---E+04	-.---E+0b	-.702E+0b
.363E-0B	-.---5E+0b	-.---7E+0b	-.---E+04	.---1E+0b	.---5E+0b	.---7E+0b	.---E+0b	-.---1E+0b
.375E-0B	-.---2E+0b	-.---7E+0b	-.---1E+0b	-.13-E+05	.---2E+0b	.---7E+0b	.---1E+0b	.13-E+05
.388E-0B	.29bE+05	-.---7E+0b	-.---9E+0b	-.29bE+05	.---7E+0b	.---E+0b	.---2E+0b	.---9E+0b
.---E-0B	.---E+0b	.3--E+05	-.---3E+0b	-.---E+0b	-.---E+0b	-.3--E+05	.---3E+0b	.---E+0b
.413E-0B	.-70E+0b	.5-3E+0b	.---5E+05	-.---E+0b	-.-70E+0b	-.503E+0b	-.---5E+05	.---E+0b
.---E-0B	.---E+0b	.---7E+0b	.502E+0b	.---5E+05	-.---E+0b	-.---7E+0b	-.502E+0b	-.---5E+05
.---E-0B	-.525E+05	.---3E+0b	.---E+0b	.507E+0b	.525E+05	-.---3E+0b	-.---E+0b	-.507E+0b
.---E-0B	-.519E+0b	-.---7E+05	.---9E+0b	.---0E+0b	.519E+0b	.---7E+05	-.---9E+0b	-.---0E+0b
.---E-0B	-.---5E+0b	-.531E+0b	-.---8E+05	.---5E+0b	.---3E+0b	.531E+0b	.---7E+05	-.---5E+0b
.---E-0B	-.---5E+0b	-.---2E+0b	-.53-E+0b	-.---2E+05	.---5E+0b	.53-E+0b	.---2E+0b	.---2E+05
.---E-0B	.572E+05	-.---1E+0b	-.---5E+0b	-.---2E+0b	.572E+05	.---1E+0b	.---5E+0b	.---2E+0b
.---E-0B	.531E+0b	.---2E+05	-.---1E+0b	-.---4E+0b	-.531E+0b	-.---2E+05	.---1E+0b	.---4E+0b

TIME	UPSTREAM COMPONENT OF PRESSURE							
0.	.500E+06	.354E+06	-.858E-09	-.354E+06	-.500E+06	-.354E+06	-.809E-08	.354E+06
.125E+04	.327E+06	.475E+06	.345E+06	.123E+05	-.327E+06	-.475E+06	-.345E+06	-.123E+05
.250E+04	-.119E+05	.327E+06	.474E+06	.343E+06	.119E+05	-.327E+06	-.474E+06	-.343E+06
.375E+04	-.349E+06	-.187E+05	.326E+06	.477E+06	.349E+06	.187E+05	-.326E+06	-.477E+06
.500E+04	-.480E+06	-.352E+06	-.180E+05	.327E+06	.480E+06	.352E+06	.180E+05	-.327E+06
.625E+04	-.333E+06	-.486E+06	-.354E+06	-.150E+05	.333E+06	.486E+06	.354E+06	.150E+05
.750E+04	.112E+05	-.336E+06	-.487E+06	-.352E+06	-.112E+05	.336E+06	.487E+06	.352E+06
.875E+04	.347E+06	.650E+04	-.338E+06	-.485E+06	-.347E+06	-.650E+04	.338E+06	.485E+06
.100E+05	.482E+06	.344E+06	.547E+04	-.337E+06	-.482E+06	-.344E+06	-.547E+04	.337E+06
.113E+05	.334E+06	.479E+06	.343E+06	.891E+04	-.334E+06	-.479E+06	-.343E+06	-.891E+04
.125E+05	-.808E+04	.333E+06	.478E+06	.344E+06	.808E+04	-.333E+06	-.478E+06	-.344E+06
.138E+05	-.345E+06	-.918E+04	.332E+06	.479E+06	.345E+06	.918E+04	-.332E+06	-.479E+06
.150E+05	-.479E+06	-.345E+06	-.909E+04	.332E+06	.479E+06	.345E+06	.909E+04	-.332E+06
.163E+05	-.332E+06	-.478E+06	-.345E+06	-.922E+04	.332E+06	.478E+06	.345E+06	.922E+04
.175E+05	.110E+05	-.331E+06	-.478E+06	-.345E+06	-.100E+05	.331E+06	.478E+06	.345E+06
.188E+05	.345E+06	.107E+05	-.331E+06	-.479E+06	-.346E+06	-.107E+05	.331E+06	.478E+06
.200E+05	.479E+06	.347E+06	.109E+05	-.331E+06	-.479E+06	-.347E+06	-.109E+05	.331E+06
.213E+05	.332E+06	.480E+06	.347E+06	.108E+05	-.332E+06	-.480E+06	-.347E+06	-.108E+05
.225E+05	-.107E+05	.332E+06	.480E+06	.347E+06	.107E+05	-.332E+06	-.480E+06	-.347E+06
.238E+05	-.347E+06	-.110E+05	.332E+06	.480E+06	.347E+06	.110E+05	-.332E+06	-.480E+06
.250E+05	-.480E+06	-.346E+06	-.112E+05	.332E+06	.480E+06	.346E+06	.112E+05	-.332E+06
.263E+05	-.332E+06	-.480E+06	-.346E+06	-.112E+05	.332E+06	.480E+06	.346E+06	.112E+05
.275E+05	.112E+05	-.332E+06	-.480E+06	-.346E+06	-.112E+05	.332E+06	.480E+06	.346E+06
.288E+05	.346E+06	.112E+05	-.332E+06	-.480E+06	-.346E+06	-.112E+05	.332E+06	.480E+06
.300E+05	.480E+06	.346E+06	.112E+05	-.332E+06	-.480E+06	-.346E+06	-.112E+05	.332E+06
.313E+05	.332E+06	.480E+06	.346E+06	.112E+05	-.332E+06	-.480E+06	-.346E+06	-.112E+05
.325E+05	-.112E+05	.332E+06	.480E+06	.346E+06	.112E+05	-.332E+06	-.480E+06	-.346E+06
.338E+05	-.346E+06	-.112E+05	.332E+06	.480E+06	.346E+06	.112E+05	-.332E+06	-.480E+06
.350E+05	-.480E+06	-.346E+06	-.112E+05	.332E+06	.480E+06	.346E+06	.112E+05	-.332E+06
.363E+05	-.332E+06	-.480E+06	-.346E+06	-.112E+05	.332E+06	.480E+06	.346E+06	.112E+05
.375E+05	.112E+05	-.332E+06	-.480E+06	-.346E+06	-.112E+05	.332E+06	.480E+06	.346E+06
.388E+05	.346E+06	.112E+05	-.332E+06	-.480E+06	-.346E+06	-.112E+05	.332E+06	.480E+06
.400E+05	.480E+06	.346E+06	.112E+05	-.332E+06	-.480E+06	-.346E+06	-.112E+05	.332E+06
.413E+05	.332E+06	.480E+06	.346E+06	.112E+05	-.332E+06	-.480E+06	-.346E+06	-.112E+05
.425E+05	-.112E+05	.332E+06	.480E+06	.346E+06	.112E+05	-.332E+06	-.480E+06	-.346E+06
.438E+05	-.346E+06	-.112E+05	.332E+06	.480E+06	.346E+06	.112E+05	-.332E+06	-.480E+06
.450E+05	-.480E+06	-.346E+06	-.112E+05	.332E+06	.480E+06	.346E+06	.112E+05	-.332E+06
.463E+05	-.332E+06	-.480E+06	-.346E+06	-.112E+05	.332E+06	.480E+06	.346E+06	.112E+05
.475E+05	.112E+05	-.332E+06	-.480E+06	-.346E+06	-.112E+05	.332E+06	.480E+06	.346E+06
.488E+05	.346E+06	.112E+05	-.332E+06	-.480E+06	-.346E+06	-.112E+05	.332E+06	.480E+06
.500E+05	.480E+06	.346E+06	.112E+05	-.332E+06	-.480E+06	-.346E+06	-.112E+05	.332E+06

```
PARAMETERS USED
DTIME  =  .250E-05
NY     =          8
NT     =        201
MAXCUN =        100
CMAKK  =  .800E+00
RHOD   =  .100E+01
C      =  .100E+04
Y      =  .100E+00
OMEGA0 =  .500E+00


TEST   RISTAR APPROACHES ITS ASYMPTOTIC VALUE

TIME STEP   1   RISTAR =  0.000   .183   ASYMPTOTE =  2.500  1.658
TIME STEP   6   RISTAR =  -.166   .827   ASYMPTOTE =  1.675  2.489
TIME STEP  11   RISTAR =  -.575  1.353   ASYMPTOTE =   .595  2.980
TIME STEP  16   RISTAR = -1.143  1.608   ASYMPTOTE =  -.575  2.986
TIME STEP  21   RISTAR = -1.750  1.526   ASYMPTOTE = -1.658  2.500
TIME STEP  26   RISTAR = -2.262  1.102   ASYMPTOTE = -2.489  1.675
TIME STEP  31   RISTAR = -2.554   .307   ASYMPTOTE = -2.980   .595
TIME STEP  36   RISTAR = -2.537  -.475   ASYMPTOTE = -2.986  -.575
TIME STEP  41   RISTAR = -2.171 -1.369   ASYMPTOTE = -2.500 -1.658
TIME STEP  46   RISTAR = -1.479 -2.124   ASYMPTOTE = -1.675 -2.489
TIME STEP  51   RISTAR =  -.583 -2.602   ASYMPTOTE =  -.595 -2.980
TIME STEP  56   RISTAR =   .508 -2.703   ASYMPTOTE =   .575 -2.986
TIME STEP  61   RISTAR =  1.518 -2.390   ASYMPTOTE =  1.658 -2.500
TIME STEP  66   RISTAR =  2.329 -1.695   ASYMPTOTE =  2.489 -1.675
TIME STEP  71   RISTAR =  2.807  -.719   ASYMPTOTE =  2.980  -.595
TIME STEP  76   RISTAR =  2.869   .406   ASYMPTOTE =  2.986   .575
TIME STEP  81   RISTAR =  2.693  1.492   ASYMPTOTE =  2.500  1.658
TIME STEP  86   RISTAR =  1.727  2.369   ASYMPTOTE =  1.675  2.489
TIME STEP  91   RISTAR =   .682  2.898   ASYMPTOTE =   .595  2.980
TIME STEP  96   RISTAR =  -.686  2.977   ASYMPTOTE =  -.575  2.986
TIME STEP 101   RISTAR = -1.592  2.597   ASYMPTOTE = -1.658  2.500
TIME STEP 106   RISTAR = -2.468  1.807   ASYMPTOTE = -2.489  1.675
TIME STEP 111   RISTAR = -2.972   .725   ASYMPTOTE = -2.980   .595
TIME STEP 116   RISTAR = -3.021  -.883   ASYMPTOTE = -2.986  -.575
TIME STEP 121   RISTAR = -2.603 -1.626   ASYMPTOTE = -2.500 -1.658
TIME STEP 126   RISTAR = -1.778 -2.527   ASYMPTOTE = -1.675 -2.489
TIME STEP 131   RISTAR =  -.672 -3.040   ASYMPTOTE =  -.595 -2.980
TIME STEP 136   RISTAR =   .586 -3.083   ASYMPTOTE =   .575 -2.986
TIME STEP 141   RISTAR =  1.686 -2.887   ASYMPTOTE =  1.658 -2.500
TIME STEP 146   RISTAR =  2.570 -1.747   ASYMPTOTE =  2.489 -1.675
TIME STEP 151   RISTAR =  3.061  -.884   ASYMPTOTE =  2.980  -.595
TIME STEP 156   RISTAR =  3.079   .576   ASYMPTOTE =  2.986   .575
TIME STEP 161   RISTAR =  2.621  1.730   ASYMPTOTE =  2.500  1.658
TIME STEP 166   RISTAR =  1.757  2.617   ASYMPTOTE =  1.675  2.489
TIME STEP 171   RISTAR =   .620  3.101   ASYMPTOTE =   .595  2.980
TIME STEP 176   RISTAR =  -.615  3.105   ASYMPTOTE =  -.575  2.986
TIME STEP 181   RISTAR = -1.756  2.628   ASYMPTOTE = -1.658  2.500
TIME STEP 186   RISTAR = -2.626  1.746   ASYMPTOTE = -2.489  1.675
TIME STEP 191   RISTAR = -3.092   .593   ASYMPTOTE = -2.980  -.595
TIME STEP 196   RISTAR = -3.082  -.649   ASYMPTOTE = -2.986  -.575
TIME STEP 201   RISTAR = -2.597 -1.790   ASYMPTOTE = -2.500 -1.658
```

TIME	INPUT 4(TIME)							
0.	0.000	.101	.143	.101	-.000	-.101	-.143	-.101
.125E-04	-.166	.468	.827	.702	.166	-.468	-.827	-.702
.250E-04	-.575	.550	1.353	1.363	.575	-.550	-1.353	-1.363
.375E-04	-1.143	.329	1.608	1.945	1.143	-.329	-1.608	-1.945
.500E-04	-1.750	-.159	1.526	2.316	1.750	.159	-1.526	-2.316
.625E-04	-2.282	-.820	1.102	2.379	2.282	.820	-1.102	-2.379
.750E-04	-2.554	-1.525	.397	2.087	2.554	1.525	-.397	-2.087
.875E-04	-2.537	-2.130	-.475	1.458	2.537	2.130	.475	-1.458
.100E-03	-2.171	-2.503	-1.369	.567	2.171	2.503	1.369	-.567
.113E-03	-1.479	-2.508	-2.124	-.456	1.479	2.508	2.124	.456
.125E-03	-.543	-2.224	-2.602	-1.456	.543	2.224	2.602	1.456
.138E-03	.508	-1.552	-2.703	-2.271	-.508	1.552	2.703	2.271
.150E-03	1.518	-.617	-2.390	-2.764	-1.518	.617	2.390	2.764
.163E-03	2.329	.448	-1.695	-2.845	-2.329	-.448	1.695	2.845
.175E-03	2.907	1.480	-.714	-2.490	-2.907	-1.480	.714	2.490
.188E-03	2.969	2.316	.406	-1.741	-2.869	-2.316	-.406	1.741
.200E-03	2.493	2.818	1.492	-.708	-2.493	-2.818	-1.492	.708
.213E-03	1.727	2.896	2.369	.456	-1.727	-2.896	-2.369	-.456
.225E-03	.882	2.529	2.894	1.584	-.882	-2.529	-2.894	-1.584
.238E-03	-.088	1.763	2.977	2.448	.088	-1.763	-2.977	-2.448
.250E-03	-1.592	.711	2.597	2.962	1.592	-.711	-2.597	-2.962
.263E-03	-2.468	-.468	1.807	3.023	2.468	.468	-1.807	-3.023
.275E-03	-2.972	-1.589	.725	2.814	2.972	1.589	-.725	-2.814
.288E-03	-3.021	-2.478	-.483	1.795	3.021	2.478	.483	-1.795
.300E-03	-2.603	-2.991	-1.626	.690	2.603	2.991	1.626	-.690
.313E-03	-1.778	-3.044	-2.527	-.529	1.778	3.044	2.527	.529
.325E-03	-.672	-2.624	-3.048	-1.675	.672	2.624	3.048	1.675
.338E-03	.546	-1.794	-3.083	-2.566	-.546	1.794	3.083	2.566
.350E-03	1.686	-.680	-2.647	-3.064	-1.686	.680	2.647	3.064
.363E-03	2.570	.547	-1.797	-3.088	-2.570	-.547	1.797	3.088
.375E-03	3.061	1.695	-.664	-2.834	-3.061	-1.695	.664	2.834
.388E-03	3.079	2.584	.575	-1.770	-3.079	-2.584	-.576	1.770
.400E-03	2.621	3.076	1.730	-.630	-2.621	-3.076	-1.730	.630
.413E-03	1.757	3.093	2.817	.608	-1.757	-3.093	-2.817	-.608
.425E-03	.620	2.631	3.101	1.755	-.620	-2.631	-3.101	-1.755
.438E-03	-.615	1.761	3.105	2.650	.615	-1.761	-3.105	-2.650
.450E-03	-1.756	.617	2.828	3.100	1.756	-.617	-2.828	-3.100
.463E-03	-2.526	-.623	1.746	3.091	2.526	.623	-1.746	-3.091
.475E-03	-3.092	-1.767	.593	2.806	3.092	1.767	-.593	-2.806
.488E-03	-3.382	-2.638	-.649	1.720	3.082	2.638	.649	-1.720
.500E-03	-2.597	-3.102	-1.790	.570	2.597	3.102	1.790	-.570

TIME	INPUT 3(TIME)							
0.	1.000	.707	-.000	-.707	-1.000	-.707	-.000	.707
.125E-04	.924	.924	.383	.383	.924	.924	.383	.383
.250E-04	.707	1.000	.707	.000	-.707	-1.000	-.707	-.000
.375E-04	.383	.924	.924	.383	-.383	-.924	-.924	-.383
.500E-04	.000	.707	1.000	.707	-.000	-.707	-1.000	-.707
.625E-04	-.383	.383	.924	.924	.383	-.383	-.924	-.924
.750E-04	-.707	.000	.707	1.000	.707	-.000	-.707	-1.000
.875E-04	-.924	-.383	.383	.924	.924	.383	-.383	-.924
.100E-03	-1.000	-.707	.000	.707	1.000	.707	.000	-.707
.113E-03	-.924	-.924	-.383	.383	.924	.924	.383	-.383
.125E-03	-.707	-1.000	-.707	.000	.707	1.000	.707	.000
.138E-03	-.383	-.924	-.924	-.383	.383	.924	.924	.383
.150E-03	-.000	-.707	-1.000	-.707	.000	.707	1.000	.707
.163E-03	.383	-.383	-.924	-.924	-.383	.383	.924	.924
.175E-03	.707	-.000	-.707	-1.000	-.707	.000	.707	1.000
.188E-03	.924	.383	-.383	-.924	-.924	-.383	.383	.924
.200E-03	1.000	.707	-.000	-.707	-1.000	-.707	.000	.707
.213E-03	.924	.924	.383	-.383	-.924	-.924	-.383	.383
.225E-03	.707	1.000	.707	-.000	-.707	-1.000	-.707	.000
.238E-03	.383	.924	.924	.383	-.383	-.924	-.924	-.383
.250E-03	.000	.707	1.000	.707	-.000	-.707	-1.000	-.707
.263E-03	-.383	.383	.924	.924	.383	-.383	-.924	-.924
.275E-03	-.707	.000	.707	1.000	.707	-.000	-.707	-1.000
.288E-03	-.924	-.383	.383	.924	.924	.383	-.383	-.924
.300E-03	-1.000	-.707	-.000	.707	1.000	.707	-.000	-.707
.313E-03	-.924	-.924	-.383	.383	.924	.924	.383	-.383
.325E-03	-.707	-1.000	-.707	-.000	.707	1.000	.707	-.000
.338E-03	-.383	-.924	-.924	-.383	.383	.924	.924	.383
.350E-03	-.000	-.707	-1.000	-.707	-.000	.707	1.000	.707
.363E-03	.383	-.383	-.924	-.924	-.383	.383	.924	.924
.375E-03	.707	-.000	-.707	-1.000	-.707	-.000	.707	1.000
.388E-03	.924	.383	-.383	-.924	-.924	-.383	.383	.924
.400E-03	1.000	.707	-.000	-.707	-1.000	-.707	.000	.707
.413E-03	.924	.924	.383	-.383	-.924	-.924	-.383	.383
.425E-03	.707	1.000	.707	-.000	-.707	-1.000	-.707	.000
.438E-03	.383	.924	.924	.383	-.383	-.924	-.924	-.383
.450E-03	.000	.707	1.000	.707	-.000	-.707	-1.000	-.707
.463E-03	-.383	.383	.924	.924	.383	-.383	-.924	-.924
.475E-03	-.707	.000	.707	1.000	.707	-.000	-.707	-1.000
.488E-03	-.924	-.383	.383	.924	.924	.383	-.383	-.924
.500E-03	-1.000	-.707	-.000	.707	1.000	.707	-.000	-.707

TIME	GREENS FUNCTION MA				
0.000	25.405	-.000	-.000	-.000	0.000
.157	-1.591	.078	.284	.317	.324
.314	-2.535	.185	.378	.388	.404
.471	-2.930	.332	.389	.395	.397
.628	-2.948	.508	.341	.318	.314
.785	-2.628	.678	.240	.194	.185
.942	-1.999	.804	.111	.060	.050
1.100	-1.525	.855	-.016	-.049	-.055
1.257	-1.291	.815	-.113	-.108	-.107
1.414	-.731	.688	-.150	-.111	-.104
1.571	-.505	.497	-.146	-.069	-.058
1.728	-.395	.290	-.077	-.007	.003
1.885	-.367	.076	.030	.051	.054
2.042	-.379	-.083	.153	.082	.076
2.199	-.392	-.180	.267	.077	.064
2.356	-.384	-.213	.351	.040	.028
2.513	-.348	-.197	.390	-.011	-.016
2.670	-.293	-.155	.382	-.056	-.047
2.827	-.236	-.110	.332	-.078	-.054
2.985	-.191	-.078	.255	-.066	-.036
3.142	-.167	-.067	.167	-.018	.003
3.299	-.162	-.075	.085	.052	.039
3.456	-.170	-.091	.020	.126	.059
3.613	-.179	-.106	-.020	.189	.053
3.770	-.181	-.111	-.037	.228	.021
3.927	-.172	-.103	-.036	.237	-.023
4.084	-.153	-.087	-.025	.219	-.061
4.241	-.132	-.067	-.013	.183	-.075
4.398	-.114	-.051	-.006	.139	-.051
4.555	-.104	-.044	-.006	.097	.011
4.712	-.104	-.045	-.014	.061	.100
4.869	-.110	-.052	-.026	.036	.197
5.027	-.117	-.060	-.037	.015	.280
5.184	-.120	-.064	-.043	.002	.329
5.341	-.116	-.060	-.043	-.007	.336
5.498	-.106	-.051	-.036	-.011	.303
5.655	-.093	-.040	-.027	-.008	.242
5.812	-.083	-.030	-.017	.004	.170
5.969	-.078	-.025	-.012	.024	.104
6.126	-.079	-.026	-.012	.050	.056
6.283	-.085	-.032	-.018	.077	.031
6.440	-.091	-.038	-.028	.099	.025
6.597	-.094	-.042	-.037	.111	.030
6.754	-.092	-.041	-.042	.110	.037
6.912	-.084	-.034	-.039	.095	.039
7.069	-.075	-.025	-.027	.072	.034
7.226	-.067	-.016	-.008	.046	.022
7.383	-.064	-.011	.017	.022	.008
7.540	-.066	-.013	.041	.006	-.004
7.697	-.072	-.020	.061	-.001	-.010
7.854	-.079	-.029	.073	-.000	-.009

TIME	GREENS FUNCTION MA				
8.011	-.083	-.036	.075	.004	-.005
8.168	-.081	-.037	.069	.009	.000
8.325	-.074	-.030	.055	.010	.003
8.482	-.066	-.014	.038	.008	.002
8.639	-.055	.006	.021	.002	-.003
8.796	-.052	.027	.009	-.005	-.009
8.954	-.057	.045	.001	-.011	-.014
9.111	-.067	.056	-.001	-.013	-.016
9.268	-.079	.059	.000	-.012	-.015
9.425	-.087	.055	.003	-.008	-.011
9.582	-.085	.046	.005	-.005	-.008
9.739	-.071	.035	.006	-.003	-.005
9.896	-.044	.024	.004	-.003	-.005
10.053	-.009	.015	-.000	-.006	-.006
10.210	.025	.009	-.005	-.010	-.012
10.367	.053	.003	-.008	-.014	-.015
10.524	.067	-.000	-.010	-.015	-.016
10.681	.066	-.003	-.010	-.014	-.015
10.838	.052	-.003	-.007	-.010	-.012
10.996	.029	.000	-.004	-.007	-.008
11.153	.005	.002	-.002	-.005	-.006
11.310	-.015	.003	-.001	-.005	-.006
11.467	-.031	.001	-.003	-.020	-.026
11.624	-.021	.048	-.007	-.019	-.024
11.781	-.026	.052	-.011	-.018	-.020
11.938	-.026	.053	-.015	-.016	-.017
12.095	-.024	.050	-.015	-.015	-.016
12.252	-.021	.045	-.012	-.014	-.016
12.409	-.020	.038	-.005	-.014	-.019
12.566	-.021	.031	.005	-.015	-.021
12.723	-.024	.024	.016	-.017	-.024
12.881	-.029	.019	.027	-.019	-.025
13.038	-.034	.015	.034	-.020	-.025
13.195	-.037	.013	.038	-.021	-.023
13.352	-.038	.013	.021	-.020	-.020
13.509	-.037	.013	.020	-.018	-.013
13.666	-.036	.013	.017	-.013	-.005
13.823	-.032	.013	.013	-.007	-.000
13.980	-.033	.013	.008	.001	-.012
14.137	-.032	.012	.003	.008	-.016
14.294	-.031	.011	-.001	.015	-.019
14.451	-.029	.009	-.005	.020	-.020
14.608	-.028	.006	-.008	.022	-.018
14.765	-.027	.003	-.009	.022	-.012
14.923	-.027	.000	-.010	.019	-.003
15.080	-.028	.000	-.009	.014	.007
15.237	-.030	.000	-.008	.007	.020
15.394	-.032	.000	-.007	.001	.031
15.551	-.034	.000	-.006	-.005	.042
15.708	-.035	.000	-.006	-.009	.049

TIME	GREENS FUNCTION		JR
0.000	-3.698	-1.520	-.629
.157	-3.114	-1.248	-.514
.314	-2.208	-.851	-.348
.471	-1.138	-.420	-.171
.628	-.078	-.054	-.024
.785	.620	.185	.072
.942	1.048	.264	.098
1.100	1.764	.198	.073
1.257	1.788	.050	.020
1.414	1.590	-.108	-.032
1.571	1.287	-.208	-.059
1.728	.912	-.203	-.055
1.885	.601	-.090	-.024
2.042	.375	.105	.017
2.199	.242	.329	.046
2.356	.184	.525	.050
2.513	.172	.650	.024
2.670	.174	.681	-.020
2.827	.170	.625	-.062
2.985	.152	.510	-.080
3.142	.123	.373	-.056
3.299	.091	.249	.012
3.456	.065	.163	.114
3.613	.051	.121	.228
3.770	.048	.113	.325
3.927	.052	.122	.383
4.084	.055	.131	.390
4.241	.054	.128	.348
4.398	.048	.111	.273
4.555	.037	.085	.188
4.712	.027	.059	.117
4.869	.020	.043	.075
5.027	.019	.039	.066
5.184	.021	.046	.082
5.341	.025	.055	.108
5.498	.027	.06.	.125
5.655	.026	.058	.120
5.812	.021	.045	.089
5.969	.014	.029	.036
6.126	.009	.016	-.027
6.283	.007	.011	-.086
6.440	.009	.017	-.127
6.597	.013	.031	-.145
6.754	.018	.045	-.139
6.912	.019	.051	-.116
7.069	.015	.042	-.086
7.226	.009	.016	-.057
7.383	.003	-.023	-.037
7.540	-.001	-.066	-.028
7.697	.001	-.103	-.027
7.854	.008	-.125	-.030

TIME	GREENS FUNCTION		JR
8.011	.018	-.129	-.033
8.168	.024	-.116	-.033
8.325	.023	-.093	-.029
8.482	.011	-.066	-.023
8.639	-.013	-.044	-.017
8.796	-.045	-.032	-.013
8.954	-.077	-.029	-.012
9.111	-.101	-.032	-.014
9.268	-.113	-.037	-.016
9.425	-.109	-.039	-.016
9.582	-.093	-.037	-.016
9.739	-.069	-.030	-.013
9.896	-.046	-.022	-.010
10.053	-.029	-.016	-.008
10.210	-.023	-.014	-.007
10.367	-.027	-.016	-.007
10.524	-.037	-.020	-.009
10.681	-.046	-.023	-.010
10.838	-.049	-.024	-.011
10.996	-.041	-.020	-.009
11.153	-.024	-.015	-.007
11.310	.000	-.009	-.005
11.467	.019	-.015	-.010
11.624	.037	-.020	-.013
11.781	.050	-.023	-.013
11.938	.057	-.023	-.011
12.095	.056	-.020	-.006
12.252	.048	-.012	-.001
12.409	.037	-.002	.002
12.566	.025	.011	.003
12.723	.015	.024	.001
12.881	.008	.036	-.004
13.038	.006	.045	-.010
13.195	.007	.050	-.015
13.352	.009	.051	-.018
13.509	.009	.048	-.018
13.666	.005	.042	-.006
13.823	.008	.034	.016
13.980	.011	.026	.025
14.137	.013	.019	.033
14.294	.014	.014	.039
14.451	.013	.011	.042
14.608	.011	.011	.042
14.765	.008	.012	.041
14.923	.005	.014	.039
15.080	.003	.016	.035
15.237	.001	.017	.032
15.394	.001	.016	.028
15.551	.001	.015	.025
15.708	.003	.012	.022

TIME	GREENS FUNCTION AR				
0.000	25.005	-.000	-.000	-.000	0.000
.157	-.187	.009	.032	.035	.036
.314	-.256	.020	.041	.046	.045
.471	-.292	.037	.043	.048	.048
.628	-.294	.056	.038	.035	.035
.785	-.268	.075	.027	.022	.021
.942	-.222	.089	.012	.007	.006
1.100	-.169	.095	-.002	-.005	-.006
1.257	-.120	.091	-.013	-.012	-.012
1.414	-.081	.078	-.018	-.012	-.012
1.571	-.056	.055	-.016	-.008	-.006
1.728	-.048	.031	-.009	-.001	.000
1.885	-.041	.008	.003	.006	.006
2.042	-.042	-.009	.017	.009	.008
2.199	-.044	-.020	.030	.009	.007
2.356	-.043	-.024	.039	.004	.003
2.513	-.039	-.022	.043	-.001	-.002
2.670	-.033	-.017	.042	-.006	-.005
2.827	-.026	-.012	.037	-.009	-.006
2.985	-.021	-.009	.028	-.007	-.004
3.142	-.019	-.007	.019	-.002	.000
3.299	-.019	-.008	.009	.006	.004
3.456	-.019	-.010	.002	.014	.007
3.613	-.020	-.012	-.002	.021	.006
3.770	-.020	-.012	-.004	.025	.002
3.927	-.019	-.011	-.004	.026	-.003
4.084	-.017	-.010	-.003	.024	-.007
4.241	-.015	-.007	-.001	.020	-.008
4.398	-.013	-.006	-.001	.015	-.006
4.555	-.012	-.005	-.001	.011	-.001
4.712	-.012	-.005	-.002	.007	.011
4.869	-.012	-.006	-.003	.004	.022
5.027	-.013	-.007	-.004	.002	.031
5.184	-.013	-.007	-.005	.000	.037
5.341	-.013	-.007	-.005	-.001	.037
5.498	-.012	-.006	-.004	-.001	.034
5.655	-.010	-.004	-.003	-.001	.027
5.812	-.009	-.003	-.002	.000	.019
5.969	-.009	-.003	-.001	.003	.012
6.126	-.009	-.003	-.001	.006	.006
6.283	-.009	-.004	-.002	.009	.003
6.440	-.010	-.004	-.003	.011	.003
6.597	-.010	-.005	-.004	.012	.003
6.754	-.010	-.005	-.005	.012	.004
6.912	-.009	-.004	-.004	.011	.004
7.069	-.008	-.003	-.003	.008	.004
7.226	-.007	-.002	-.001	.005	.002
7.383	-.007	-.001	.002	.002	.001
7.540	-.007	-.001	.005	.001	-.000
7.697	-.008	-.002	.007	-.000	-.001
7.854	-.009	-.003	.008	-.000	-.001

TIME	GREENS FUNCTION AR				
8.011	-.009	-.004	.008	.000	-.001
8.168	-.009	-.004	.008	.001	.000
8.325	-.008	-.003	.006	.001	.000
8.482	-.007	-.002	.004	.001	.000
8.639	-.006	.001	.002	.000	-.000
8.796	-.006	.003	.001	-.001	-.001
8.954	-.006	.005	.000	-.001	-.002
9.111	-.007	.006	-.000	-.001	-.002
9.268	-.009	.007	.000	-.001	-.002
9.425	-.010	.006	.000	-.001	-.001
9.582	-.009	.005	.001	-.001	-.001
9.739	-.008	.004	.001	-.000	-.001
9.896	-.005	.003	.000	-.000	-.001
10.053	-.001	.002	-.000	-.001	-.001
10.210	.003	.001	-.001	-.001	-.001
10.367	.006	.000	-.001	-.002	-.002
10.524	.007	-.000	-.001	-.002	-.002
10.681	.007	-.000	-.001	-.002	-.002
10.838	.006	-.000	-.001	-.001	-.001
10.996	.003	.000	-.000	-.001	-.001
11.153	.001	.001	-.000	-.001	-.001
11.310	-.002	.002	-.000	-.001	-.001
11.467	-.001	.005	-.000	-.002	-.003
11.624	-.002	.005	-.001	-.002	-.003
11.781	-.003	.006	-.001	-.002	-.002
11.938	-.003	.006	-.002	-.002	-.002
12.095	-.003	.006	-.002	-.002	-.002
12.252	-.002	.005	-.001	-.002	-.002
12.409	-.002	.004	-.001	-.002	-.002
12.566	-.002	.003	.001	-.002	-.002
12.723	-.003	.003	.002	-.002	-.003
12.881	-.003	.002	.003	-.002	-.003
13.038	-.004	.002	.004	-.002	-.003
13.195	-.004	.001	.004	-.002	-.003
13.352	-.003	.001	.002	-.002	-.000
13.509	-.003	.001	.002	-.002	-.000
13.666	-.003	.001	.002	-.001	-.001
13.823	-.004	.001	.001	-.001	-.001
13.980	-.004	.001	.001	.000	-.001
14.137	-.004	.001	.000	.001	-.002
14.294	-.003	.001	-.000	.002	-.002
14.451	-.003	.001	-.001	.002	-.002
14.608	-.003	.001	-.001	.002	-.002
14.765	-.003	.001	-.001	.002	-.001
14.923	-.003	.001	-.001	.002	-.000
15.080	-.003	.001	-.001	.002	.001
15.237	-.003	.001	-.001	.001	.002
15.394	-.004	.001	-.001	.000	.003
15.551	-.004	.001	-.001	-.001	.005
15.708	-.004	.001	-.001	-.001	.005

TIME	PRESSURE P(TIME) FOR DISCRETE VALUES OF r							
0.	.526E+06	.372E+06	-.744E-09	-.372E+06	-.526E+06	-.372E+06	-.822E-05	.372E+06
.125E+04	.542E+06	.528E+06	.205E+06	-.238E+06	-.542E+06	-.528E+06	-.205E+06	.238E+06
.250E+04	.620E+06	.741E+06	.427E+06	-.135E+06	-.620E+06	-.741E+06	-.427E+06	.135E+06
.375E+04	.644E+06	.766E+06	.644E+06	.144E+06	-.644E+06	-.766E+06	-.644E+06	-.144E+06
.500E+04	.926E+05	.599E+06	.755E+06	.486E+06	-.926E+05	-.599E+06	-.755E+06	-.486E+06
.625E+04	-.377E+06	.235E+06	.709E+06	.767E+06	.377E+06	-.235E+06	-.709E+06	-.767E+06
.750E+04	-.859E+06	-.256E+06	.476E+06	.930E+06	.859E+06	.256E+06	-.476E+05	-.930E+06
.875E+04	-.121E+07	-.709E+06	.709E+05	.911E+06	.121E+07	.709E+06	-.709E+05	-.911E+06
.100E+05	-.139E+07	-.128E+07	-.427E+06	.673E+06	.139E+07	.128E+07	.427E+06	-.673E+06
.113E+05	-.151E+07	-.160E+07	-.955E+06	.252E+06	.131E+07	.160E+07	.955E+06	-.252E+06
.125E+05	-.963E+06	-.169E+07	-.141E+07	-.301E+06	.963E+06	.169E+07	.141E+07	.301E+06
.138E+05	-.447E+06	-.151E+07	-.169E+07	-.880E+06	.447E+06	.151E+07	.169E+07	.880E+06
.150E+05	.222E+06	-.167E+07	-.174E+07	-.139E+07	-.222E+06	.107E+07	.174E+07	.139E+07
.163E+05	.904E+06	-.433E+06	-.152E+07	-.171E+07	-.904E+06	.433E+06	.152E+07	.171E+07
.175E+05	.149E+07	.312E+06	-.104E+07	-.179E+07	-.149E+07	-.312E+06	.104E+07	.179E+07
.188E+05	.185E+07	.104E+07	-.384E+06	-.159E+07	-.185E+07	-.104E+07	.384E+06	.159E+07
.200E+05	.144E+07	.163E+07	.349E+06	-.111E+07	-.194E+07	-.163E+07	-.349E+06	.111E+07
.213E+05	.172E+07	.199E+07	.139E+07	-.439E+06	-.172E+07	-.199E+07	-.139E+07	.439E+06
.225E+05	.123E+07	.207E+07	.167E+07	.336E+06	-.120E+07	-.203E+07	-.167E+07	-.336E+06
.238E+05	.47CE+06	.175E+07	.201E+07	.109E+07	-.470E+06	-.175E+07	-.201E+07	-.109E+07
.250E+05	-.359E+06	.118E+07	.203E+07	.169E+07	.355E+06	-.118E+07	-.203E+07	-.169E+07
.263E+05	-.114E+07	.419E+06	.173E+07	.203E+07	.114E+07	-.419E+06	-.173E+07	-.203E+07
.275E+05	-.176E+07	-.425E+06	.116E+07	.206E+07	.176E+07	.425E+06	-.116E+07	-.206E+07
.288E+05	-.211E+07	-.122E+07	.391E+06	.177E+07	.211E+07	.122E+07	-.391E+06	-.177E+07
.300E+05	-.213E+07	-.193E+07	-.453E+06	.119E+07	.213E+07	.193E+07	.453E+06	-.119E+07
.313E+05	-.192E+07	-.210E+07	-.124E+07	.412E+06	.192E+07	.210E+07	.124E+07	-.412E+06
.325E+05	-.172E+07	-.217E+07	-.174E+07	-.441E+06	.122E+07	.217E+07	.174E+07	.441E+06
.338E+05	-.141E+06	-.185E+07	-.217E+07	-.124E+07	.419E+06	.185E+07	.217E+07	.124E+07
.350E+05	.456E+06	-.121E+07	-.210E+07	-.185E+07	-.456E+06	.121E+07	.210E+07	.135E+07
.363E+05	.127E+07	-.392E+06	-.182E+07	-.214E+07	-.127E+07	.392E+06	.182E+07	.214E+07
.375E+05	.194E+07	.490E+06	-.119E+07	-.214E+07	-.189E+07	-.490E+06	.119E+07	.214E+07
.388E+05	.222E+07	.130E+07	-.378E+06	-.184E+07	-.222E+07	-.130E+07	.378E+06	.184E+07
.400E+05	.201E+07	.191E+07	.501E+06	-.121E+07	-.221E+07	-.191E+07	.501E+06	.121E+07
.413E+05	.159E+07	.223E+07	.131E+07	-.393E+06	-.185E+07	-.223E+07	-.131E+07	.393E+06
.425E+05	.121E+07	.221E+07	.192E+07	.503E+06	-.122E+07	-.221E+07	-.192E+07	-.503E+06
.438E+05	.376E+06	.184E+07	.223E+07	.131E+07	-.376E+06	-.184E+07	-.223E+07	-.131E+07
.450E+05	-.574E+06	.111E+07	.220E+07	.193E+07	.524E+06	-.111E+07	-.220E+07	-.193E+07
.463E+05	-.114E+07	.547E+06	.183E+07	.224E+07	.134E+07	-.547E+06	-.183E+07	-.224E+07
.475E+05	-.195E+07	-.540E+06	.111E+07	.221E+07	.195E+07	.540E+06	-.111E+07	-.221E+07
.488E+05	-.222E+07	-.134E+07	.340E+06	.184E+07	.224E+07	.134E+07	-.340E+06	-.184E+07
.500E+05	-.222E+07	-.196E+07	-.550E+06	.111E+07	.222E+07	.196E+07	.550E+06	-.111E+07

TIME	UPSTREAM COMPONENT OF PRESSURE							
0.	.500E+06	.354E+06	-.85PE-09	-.354E+06	-.500E+06	-.354E+06	-.809E-08	.354E+06
.125E-04	.434E+06	.434E+06	.187E+06	-.175E+06	-.434E+06	-.434E+06	-.187E+06	.175E+06
.25JE-0.	.330E+06	.472E+06	.338E+06	.595E+04	-.330E+06	-.472E+06	-.338E+06	-.595E+04
.375E-04	.169E+06	.429E+06	.438E+06	.190E+06	-.169E+06	-.429E+06	-.438E+06	-.190E+06
.500E-04	-.145E+05	.321E+06	.489E+06	.342E+06	.145E+05	-.321E+06	-.489E+06	-.342E+06
.625E-04	-.199E+06	.181E+06	.427E+06	.442E+06	.199E+06	-.181E+06	-.427E+06	-.442E+06
.75JE-04	-.351E+06	-.240E+05	.317E+06	.473E+06	.351E+06	.240E+05	-.317E+06	-.473E+06
.875E-04	-.451E+06	-.207E+06	.15PE+06	.431E+06	.451E+06	.207E+06	-.15PE+06	-.431E+06
.100E-03	-.500E+06	-.354E+06	-.273E+05	.320E+06	.480E+06	.359E+06	.273E+05	-.320E+06
.113E-03	-.437E+06	-.457E+06	-.210E+06	.181E+06	.437E+06	.457E+06	.210E+06	-.181E+06
.125E-03	-.324E+06	-.485E+06	-.3o1E+06	.258E+05	.324E+06	.485E+06	.3o1E+06	.25PE+05
.138E-03	-.183E+06	-.439E+06	-.458E+06	-.209E+06	.183E+06	.439E+06	.458E+06	.209E+06
.15JE-03	.25iE+05	-.325E+06	-.485E+06	-.3o1E+06	-.251E+05	.325E+06	.485E+06	.3o1E+06
.163E-03	.209E+06	-.182E+06	-.439E+06	-.459E+06	-.209E+06	.182E+06	.439E+06	.458E+06
.175E-03	.3o2E+06	.2o0E+05	-.325E+05	-.48oE+06	-.3o2E+06	-.2o0E+05	.325E+06	.48oE+06
.188E-03	.459E+06	.210E+06	-.182E+06	-.439E+06	-.459E+06	-.21oE+06	.182E+06	.439E+06
.2..E-03	.48?E+06	.3o3E+06	.265E+05	-.32oE+06	-.48?E+06	-.3o3E+06	-.265E+05	.32oE+06
.213E-03	.44.E+06	.45iE+06	.211E+06	-.182E+06	-.44oE+06	-.45iE+06	-.211E+06	.182E+06
.225E-03	.325E+06	.48oE+06	.3o4E+06	.2o0E+05	-.325E+06	-.48oE+06	-.3o4E+06	-.2o0E+05
.238E-03	.182E+06	.44oE+06	.45iE+06	.211E+06	-.182E+06	-.44oE+06	-.45iE+06	-.211E+06
.25oE-03	-.272E+05	.325E+06	.48?E+06	.3o4E+06	.272E+05	-.325E+06	-.48?E+06	-.3o4E+06
.263E-03	-.212E+06	.181E+06	.44oE+06	.45iE+06	.212E+06	-.181E+06	-.44oE+06	-.45iE+06
.275E-03	-.3o4E+06	-.272E+05	.325E+06	.48?E+06	.3o4E+06	.272E+05	-.325E+06	-.48?E+06
.288E-03	-.45iE+06	-.212E+06	.181E+06	.44oE+06	.45iE+06	.212E+06	-.181E+06	-.44oE+06
.3..E-03	-.48?E+06	-.3o4E+06	-.272E+05	.325E+06	.48?E+06	.3o4E+06	.272E+05	-.325E+06
.313E-03	-.44oE+06	-.45iE+06	-.212E+06	.181E+06	.44oE+06	.45iE+06	.212E+06	-.181E+06
.325E-03	-.325E+06	-.48?E+06	-.3o4E+06	-.272E+05	.325E+06	.48?E+06	.3o4E+06	.272E+05
.338E-03	-.181E+06	-.44oE+06	-.45iE+06	-.212E+06	.181E+06	.44oE+06	.45iE+06	.212E+06
.35oE-03	.272E+05	-.325E+06	-.48?E+06	-.3o4E+06	-.272E+05	.325E+06	.48?E+06	.3o4E+06
.363E-03	.212E+06	-.181E+06	-.44oE+06	-.45iE+06	-.212E+06	.181E+06	.44oE+06	.45iE+06
.375E-03	.3o4E+06	.272E+05	-.325E+06	-.48?E+06	-.3o4E+06	-.272E+05	.325E+06	.48?E+06
.388E-03	.45iE+06	.212E+06	-.181E+06	-.44oE+06	-.45iE+06	-.212E+06	.181E+06	.44oE+06
.4..E-03	.48?E+06	.3o4E+06	.272E+05	-.325E+06	-.48?E+06	-.3o4E+06	-.272E+05	.325E+06
.413E-03	.44oE+06	.45iE+06	.212E+06	-.181E+06	-.44oE+06	-.45iE+06	-.212E+06	.181E+06
.425E-03	.325E+06	.48?E+06	.3o4E+06	.272E+05	-.325E+06	-.48?E+06	-.3o4E+06	-.272E+05
.438E-03	.181E+06	.44oE+06	.45iE+06	.212E+06	-.181E+06	-.44oE+06	-.45iE+06	-.212E+06
.45oE-03	-.272E+05	.325E+06	.48?E+06	.3o4E+06	.272E+05	-.325E+06	-.48?E+06	-.3o4E+06
.463E-03	-.212E+06	.181E+06	.44oE+06	.45iE+06	.212E+06	-.181E+06	-.44oE+06	-.45iE+06
.475E-03	-.3o4E+06	-.272E+05	.325E+06	.48?E+06	.3o4E+06	.272E+05	-.325E+06	-.48?E+06
.488E-03	-.45iE+06	-.212E+06	.181E+06	.44oE+06	.45iE+06	.212E+06	-.181E+06	-.44oE+06
.5..E-03	-.48?E+06	-.3o4E+06	-.272E+05	.325E+06	.48?E+06	.3o4E+06	.272E+05	-.325E+06

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15 Supplementary Notes

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16 Abstract

A small perturbation type analysis has been developed for the acoustic far field in an infinite duct extending upstream and downstream of an axial turbomachinery stage. The analysis is designed to interface with the numerical solution of the near field of the blade rows described in NASA CR-2900 and thereby to provide the necessary closure condition to complete the statement of infinite duct boundary conditions for the subject problem. The present analysis differs from conventional inlet duct analyses in that a simple harmonic time dependence was not assumed, since a transient signal is generated by the numerical near-field solution and periodicity is attained only asymptotically. A description of the computer code developed to carry out the necessary convolutions numerically is included, as well as the results of a sample application using an impulsively initiated harmonic signal.

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